MATHEMATICS 201-BNK-05
Advanced Calculus
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III – Vector Valued Functions

1. Sketch the curve with the given vector equation. Indicate with an arrow the direction in qhich t increases.
   a) $\vec{r}(t)=(t^2,t)$
   b) $\vec{r}(t)=(2t,3,-t)$
   c) $\vec{r}(t)=(t,\sin t,\cos t)$
   d) $\vec{r}(t)=(\cos 2t,2,\sin 2t)$

2. Find the limit.
   a) $\lim_{t\to 4}\left(t^3-1,e^{2t+1},\frac{1}{t-1}\right)$
   b) $\lim_{t\to 0}\left(\frac{e^t-1}{t},\frac{\sqrt{t+4}-2}{t},\frac{1}{1+t}\right)$
   c) $\lim_{t\to 1}\left(\sin\frac{\pi}{2t},\frac{\ln t}{t-1},\frac{t^2-1}{t^2+5t-6}\right)$

3. Find $\vec{r}'(t)$.
   a) $\vec{r}(t)=(3-t,3t^2,\sin 3t)$
   b) $\vec{r}(t)=\left(\frac{t^2}{t^2+4},\ln\sin t,e^{\tan t}\right)$
   c) $\vec{r}(t)=e^{2t}\mathbf{i}+(3t^3-t)\mathbf{j}+\sqrt{\sec t}\,\mathbf{\kappa}$

4. Find the unit tangent vector at the point with the given value of the parameter t.
   a) $\vec{r}(t)=\left(3t^2,-2t+1,\frac{4}{t}\right)$ at $t=2$
   b) $\vec{r}(t)=(\cos 2t,\sin 3t,\tan t)$ at $t=\frac{\pi}{4}$
   c) $\vec{r}(t)=\left(\ln(9+4t),\sqrt{t^2+3},\cos^3(\pi t)\right)$ at $t=-1$

5. Find the parametric equations for the tangent line to the curve with the given value of the parameter t.
   a) $\vec{r}(t)=(3t^5-1,4t^3,2t+2)$ at $t=1$
   b) $\vec{r}(t)=(e^{-t}\cos t,e^{-t}\sin t,e^{-t})$ at $t=0$
   c) $\vec{r}(t)=\left(\ln(t-1),\frac{5t}{t^2+1},4-t^2\right)$ at $t=2$

6. Let $r(t)=(\sin t, t, \cos t)$. Find
   a) $\lim_{t\to 0}\left(\vec{r}(t)-\vec{r}'(t)\right)$
   b) $\lim_{t\to 0}\left(\vec{r}(t)\bullet\vec{r}'(t)\right)$
   c) $\lim_{t\to 0}\left(\vec{r}(t)\times\vec{r}'(t)\right)$

7. Let $\vec{r}(t)=(t,t^2,t^3)$. Find $\lim_{t\to 1}\vec{r}(t)\bullet\left(\vec{r}'(t)\times\vec{r}''(t)\right)$.

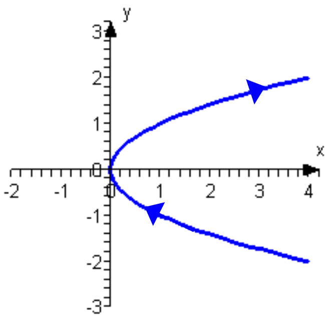
8. Evaluate the indefinite integral.
   a) $\int\left(t^3,\cos 2t,\frac{2}{t^2+1}\right)dt$
   b) $\int\left(\sqrt{t},te^t,\ln t\right)dt$

9. Evaluate the definite integral.
   a) $\int_0^{\frac{\pi}{3}}(\cos 3t,-\sin 3t,t)\,dt$
   b) $\int_1^2\left(e^t,\frac{2}{t+1},(2t-3)^3\right)dt$

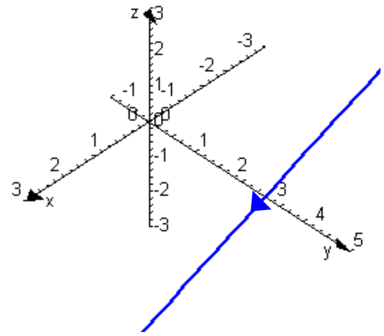
10. Using the formulas for differentiation, prove the following formula.
$$\frac{d}{dt}\left[\vec{u}\bullet(\vec{v}\times\vec{w})\right]=\frac{d\vec{u}}{dt}\bullet(\vec{v}\times\vec{w})+\vec{u}\bullet\left(\frac{d\vec{v}}{dt}\times\vec{w}\right)+\vec{u}\bullet\left(\vec{v}\times\frac{d\vec{w}}{dt}\right)$$

ANSWERS

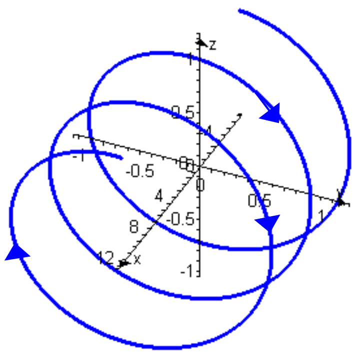
1. a)



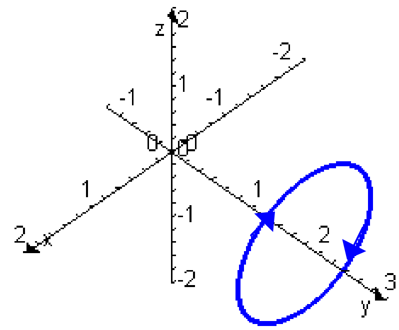
b)



c)



d)



2. a) $\left(63, e^9, \frac{1}{3}\right)$ b) $\left(1, \frac{1}{4}, 1\right)$ c) $\left(1, 1, \frac{2}{7}\right)$

3. a) $\vec{r}'(t) = (-1, 6t, 3\cos 3t)$ b) $\vec{r}'(t) = \left(\frac{8t}{(t^2+4)^2}, \cot t, e^{\tan t}\sec^2 t\right)$

 c) $\vec{r}'(t) = \left(2e^{2t}, 9t^2, \frac{1}{2}\sqrt{\sec t}\,\tan t\right)$

4. a) $\left(\frac{12\sqrt{149}}{149}, \frac{-2\sqrt{149}}{149}, \frac{-\sqrt{149}}{149}\right)$ b) $\left(\frac{-2\sqrt{2}}{5}, \frac{-3}{5}, \frac{2\sqrt{2}}{5}\right)$ c) $\left(\frac{8\sqrt{89}}{89}, \frac{-5\sqrt{89}}{89}, 0\right)$

5. a) $\begin{cases} x = 2 + 15t \\ y = 4 + 12t \\ z = 4 + 2t \end{cases}$ b) $\begin{cases} x = 1 - t \\ y = t \\ z = 1 - t \end{cases}$ c) $\begin{cases} x = t \\ y = 2 - \frac{3}{5}t \\ z = -4t \end{cases}$

6. a) $\lim\limits_{t\to 0}(\sin t - \cos t, t - 1, \sin t + \cos t) = (-1, -1, 1)$ b) $\lim\limits_{t\to 0} t = 0$

 c) $\lim\limits_{t\to 0}(-\cos t - t\sin t, 1, \sin t - t\cos t) = (-1, 1, 0)$

7. $\lim\limits_{t\to 1} 2t^3 = 2$

8. a) $\left(\frac{1}{4}t^4, \frac{1}{2}\sin 2t, 2\arctan t\right)$ b) $\left(\frac{2}{3}t^{\frac{3}{2}}, te^t - e^t, t\ln t - t\right)$

9. a) $\left(0, \frac{-2}{3}, \frac{\pi^2}{18}\right)$ b) $\left(e^2 - e, 2\ln 3 - 2\ln 2, 0\right)$

10. $$\begin{aligned} \tfrac{d}{dt}\left[\vec{u}\bullet(\vec{v}\times\vec{w})\right] &= \tfrac{d}{dt}[\vec{u}]\bullet(\vec{v}\times\vec{w}) + \vec{u}\bullet\tfrac{d}{dt}[\vec{v}\times\vec{w}] \\ &= \tfrac{d\vec{u}}{dt}\bullet(\vec{v}\times\vec{w}) + \vec{u}\bullet\left(\left(\tfrac{d\vec{v}}{dt}\times\vec{w}\right) + \vec{u}\bullet\left(\vec{v}\times\tfrac{d\vec{w}}{dt}\right)\right) \\ &= \tfrac{d\vec{u}}{dt}\bullet(\vec{v}\times\vec{w}) + \vec{u}\bullet\left(\tfrac{d\vec{v}}{dt}\times\vec{w}\right) + \vec{u}\bullet\left(\vec{v}\times\tfrac{d\vec{w}}{dt}\right) \end{aligned}$$