



## MATHEMATICS 201-BNK-05

Advanced Calculus

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# II - Continuity

1. Use the formal definition for continuity to prove that the following functions are continuous at the given point.

a)  $f(x) = 3x + 5$  at  $x = 2$

b)  $f(x) = -4x + 1$  at  $x = -5$

c)  $f(x) = 2x^2 - 3$  at  $x = 1$

d)  $f(x) = x^2 - 5x + 2$  at  $x = -2$

e)  $f(x) = \frac{3}{x}$  at  $x = -6$

f)  $f(x) = \frac{5}{x^2}$  at  $x = -1$

g)  $f(x) = \frac{4}{2x-5}$  at  $x = 2$

h)  $f(x) = 2x^3 - 1$  at  $x = -1$

2. Show that the following functions are discontinuous at the given point.

a)  $f(x) = \begin{cases} 2 & \text{if } x \leq 1 \\ 3 & \text{if } x > 1 \end{cases}$  at  $x = 1$

b)  $f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ x & \text{if } x > 2 \end{cases}$  at  $x = 2$

c)  $f(x) = \begin{cases} \frac{1}{x} & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$  at  $x = 0$

d)  $f(x) = \begin{cases} 2x + 1 & \text{if } x \leq 3 \\ 3 - x & \text{if } x > 3 \end{cases}$  at  $x = 3$

e)  $f(x) = \begin{cases} x & \text{if } x \neq -4 \\ 5 & \text{if } x = -4 \end{cases}$  at  $x = -4$

f)  $f(x) = \begin{cases} x - 4 & \text{if } x < -4 \\ 4 - x & \text{if } x \geq -4 \end{cases}$  at  $x = -4$

3. Show that the following functions are continuous everywhere.

a)  $f(x) = 5x + 2$

b)  $f(x) = x^2 - 4$

c)  $f(x) = \frac{1}{x^2}$

d)  $f(x) = \sqrt{x}$  for  $x > 0$

4. Prove the following properties. (*These proofs resemble the ones for limits*)

a) If  $f(x)$  is continuous at  $x = a$ , then  $cf(x)$  is continuous at  $x = a$ .

b) If  $f(x)$  and  $g(x)$  are continuous at  $x = a$ , then  $f(x) + g(x)$  is continuous at  $x = a$ .

c) If  $f(x)$  and  $g(x)$  are continuous at  $x = a$ , then  $f(x)g(x)$  is continuous at  $x = a$ .