MATHEMATICS 201-BNK-05
Advanced Calculus
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I – Formal Limits

1. Use the definition of a limit to prove the following statements.

a) $\lim\limits_{x\to 3}(x-7)=-4$

b) $\lim\limits_{x\to 2}(2x+3)=7$

c) $\lim\limits_{x\to 0}x^2=0$

d) $\lim\limits_{x\to 3}\left(x^2-x\right)=6$

e) $\lim\limits_{x\to 2}\frac{1}{x}=\frac{1}{2}$

f) $\lim\limits_{x\to 2}\frac{3}{2x-5}=-3$

g) $\lim\limits_{x\to 2}x^3=8$

h) $\lim\limits_{x\to 9}\sqrt{x}=3$

i) $\lim\limits_{x\to 2}\frac{x}{x^2-3}=2$

j) $\lim\limits_{x\to -2}\frac{2}{\sqrt{x^2-6x}}=\frac{1}{2}$

k) $\lim\limits_{x\to 3}\frac{x^2-x-6}{x-3}=5$

m) $\lim\limits_{x\to 3}\left(2x^2-5x+1\right)=4$

n) $\lim\limits_{x\to 2}\left|5x-3\right|=7$

o) $\lim\limits_{x\to 3}\sqrt{5x+1}=3$

2. Use the definition of a limit to show that the following statements are false.

a) $\lim\limits_{x\to 2}(2x-1)=5$

b) $\lim\limits_{x\to 5}x^2=20$

c) $\lim\limits_{x\to 1}\frac{4}{x+1}=1$

d) $\lim\limits_{x\to 3}\frac{3}{x-3}=1$

3. Determine whether the following statements are true or false, and prove your answer with the definition.

a) $\lim\limits_{x\to 2}\left(3x^2-x+1\right)=11$

b) $\lim\limits_{x\to -2}\frac{1}{x^2-4}=1$

c) $\lim\limits_{x\to 2}f(x)=1$ where $f(x)=\begin{cases}2x-1 & x\neq 2\\ 1 & x=2\end{cases}$

d) $\lim\limits_{x\to 2}f(x)=3$ where $f(x)=\begin{cases}x^2-1 & x<1\\ x-1 & x\geq 1\end{cases}$

4. Show that the given limits do not exist.

a) $\lim\limits_{x\to 0}f(x)$ where $f(x)=\begin{cases}0 & x<0\\ 1 & x\geq 0\end{cases}$

b) $\lim\limits_{x\to 0}f(x)$ where $f(x)=\begin{cases}x^2 & x\leq 0\\ x-1 & x>0\end{cases}$

5. Use the definition of a limit to prove the following statements.

a) $\lim_{x\to 2^+}(2x+1)=5$

b) $\lim_{x\to 1^-}\left(x^2+3x\right)=4$

c) $\lim_{x\to 1^-} f(x)=5$ where $f(x)=\begin{cases}2x^2-3 & x<1\\ 5x+1 & x\geq 1\end{cases}$

d) $\lim_{x\to 2^-}[\![x]\!]=1$

e) $\lim_{x\to 2^+}\sqrt{x-2}=0$

f) $\lim_{x\to 0^-}\frac{|x|}{x}=-1$

g) $\lim_{x\to -\infty}\frac{4x}{2x+1}=2$

h) $\lim_{x\to\infty}\frac{x+3}{x}=1$

i) $\lim_{x\to 5^+}\frac{4}{5-x}=-\infty$

j) $\lim_{x\to\infty}\left(x^2+x\right)=\infty$

k) $\lim_{x\to -\infty}x^2=\infty$

6. Use the definition of a limit to show that the following statements are false.

a) $\lim_{x\to 2^+}\frac{1}{x-2}=2$

b) $\lim_{x\to 1}\frac{x-1}{x^2-1}=\infty$

c) $\lim_{x\to 3^-} f(x)=-1$ where $f(x)=\begin{cases}x^2 & x<3\\ 2-x & x\geq 3\end{cases}$

d) $\lim_{x\to 0^-}\frac{1}{x^3}=\infty$

e) $\lim_{x\to\infty}e^{-x}=\infty$

f) $\lim_{x\to -\infty}\frac{x}{x+1}=-1$

g) $\lim_{x\to 4^+}\frac{3x}{x-4}=-\infty$

7. Suppose that $\lim_{x\to a} f(x)=L$ and $\lim_{x\to a} g(x)=M$. Prove the following properties of limits.

a) $\lim_{x\to a}\left[f(x)-g(x)\right]=L-M$

b) $\lim_{x\to a}x^2=a^2$

c) $\lim_{x\to a}\left[f(x)\right]^2=L^2$ *Hint*: Use the Boundness Theorem