



## MATHEMATICS 201-BNK-05

Advanced Calculus

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# Graphing Surfaces with Maple

## Surfaces in $\mathbb{R}^3$

If a surface is given as a function  $z = f(x, y)$ , then you can use the command **plot3d(function, x-domain, y-domain, options)**. For example, to plot the elliptic paraboloid  $z = x^2 + y^2$ ,

```
plot3d(x^2+y^2, x=-2..2, y=-2..2, axes=normal);
```

If the curve is given implicitly, then use the command **implicitplot3d(equation, x-domain, y-domain, z-domain, options)** from the **plots** library. For example, the hyperboloid of one sheet  $x^2 + y^2 - z^2 = 1$

```
with(plots):
```

```
implicitplot3d(x^2+y^2-z^2=1, x=-2..2, y=-2..2, z=-2..2, axes=normal);
```

To plot level curves, we use the options **style=contour** and **orientation=[-90,0]** in the 3-d graphing. For example, the level curves for the hyperbolic paraboloid  $z = x^2 - y^2$ , we have

```
> plot3d(x^2-y^2, x=-2..2, y=-2..2, style=contour, orientation=[-90,0]);
```

We can also use the command **contourplot(function, x-domain, y-domain, options)** from the **plots** library.

```
contourplot(x^2-y^2, x=-2..2, y=-2..2);
```

For surfaces that are defined parametrically, we use the command **plot3d([x(t,s), y(t,s), z(t,s)], s-domain, t-domain, options)**. For example, plotting the doughnut given by

$$\begin{cases} x = \cos(t)(3 + \cos(s)) \\ y = \sin(t)(2 + \cos(s)) \\ z = \sin(s) \end{cases}$$

```
plot3d([cos(t)*(3+cos(s)), sin(t)*(3+cos(s)), sin(s)], s=-Pi..Pi, t=-Pi..Pi, scaling=constrained);
```

## Curves with Surfaces

To sketch a curve on a surface, this can be done in the usual way for multiplots. To make it more readable, you might use the options **color=blue** and **thickness=2**, for example, to make the curve more explicit. For example, plotting the surface  $z = x^2 + y^2$  with the curve (on the surface)  $\vec{r}(t) = (2 \cos t, 2 \sin t + 1, 4 \sin t + 5)$ ,

```
surface:=plot3d(x^2+y^2, x=-3..3, y=-3..3):
```

```
curve:=spacecurve([2*cos(t), 2*sin(t)+1, 4*sin(t)+5], t=0..2*Pi, color=blue, thickness=2):
```

```
display(surface, curve);
```

Alternatively, you can use the option **style=wireframe** for the surface. This is quite useful if you are plotting more than one surface.

## Curves in polar Coordinates

To graph curves in polar coordinates we use the command with the command **polarplot** taken from the **plots** package. For example, to graph  $r = \sin 3\theta$ , we have .

```
with(plots):  
polarplot(sin(3*theta), theta = 0 .. 2*Pi);
```

## Surfaces in Cylindrical Coordinates

For surfaces in cylindrical coordinates, we use the command with the command **cylinderplot** taken from the **plots** package, where the function is given by  $r = r(\theta, z)$ . For example, the paraboloid  $z = x^2 + y^2$ , which in cylindrical coordinates is given by  $z = r^2$ , or  $r = \sqrt{z}$  can be graphs with

```
with(plots):  
cylinderplot(sqrt(z), theta = 0 .. 2*Pi, z = 0 .. 2);
```

## Surfaces in Spherical Coordinates

For surfaces in spherical coordinates, we use the command with the command **sphereplot** taken from the **plots** package, where the function is given by  $r = r(\theta, \phi)$ . For example, the sphere  $x^2 + y^2 + z^2 = 1$ , which in spherical coordinates is  $r = 1$ , is given by

```
with(plots):  
sphereplot(1, theta = 0 .. 2*Pi, phi = 0 .. Pi);
```

If the function is not in the form  $r = r(\theta, \phi)$ , then we enter it parametrically,  $[r, \theta, \phi]$ . For example, suppose we want the cone  $z^2 = x^2 + y^2$  above the  $xy$ -plane, which, in spherical coordinates, has equation  $\phi = \frac{\pi}{4}$ .

```
sphereplot([r,theta,Pi/4],r = 0 .. 3,theta = 0 .. 2*Pi);
```