MATHEMATICS 201-BNK-05
Advanced Calculus
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Differentiation with Maple

Basic Differentiation

Differentiation of functions with more than one variable works the same way as for one variable. For example, let us find the partial derivatives of $f(x,y,z)=x^2y^3z^4-z\sqrt{4-x^2-y^2}$ when *f* is defined as an expression and when *f* is defined as a function.

A. Defining f as an expression:

f :=x^2*y^3*z^4-z*sqrt(4-x^2-y^2);

To find, for example, $\frac{\partial f}{\partial y}$, we have

diff(f(x,y,z),y);

where the last letter, after the coma, represents the variable with which we differentiate.
We can also find higher order derivatives, for example $\frac{\partial^3 f}{\partial y\partial y\partial x}$.

diff(f(x,y,z),x,y,y);

B: Using the functional notation

f := (x,y,z)-> x^2*y^3*z^4-z*sqrt(4-x^2-y^2);

We use the command "D" to find $\frac{\partial f}{\partial y}$

D[2](f);

where the 2 means that we differentiate with respect to the second variable *y*.
To evaluate the derivative $\frac{\partial^2 f}{\partial y\partial x}$ at the point (1,1,1), we just add it after *f*.

D[2,1](f)(1,1,1);

Chain Rule

Consider the function $f(x,y)=\sin(x)\cos(y)$ where *u* and *v* are functions of *x* and *y* given by $x=u^2+v^2$ and $y=uv$. Let us find $\frac{\partial f}{\partial u}$ and $\frac{\partial f}{\partial v}$.

f := (x,y)->sin(x)*cos(y);
x := u^2+v^2;
y:=u*v;

The partial derivatives with respect to *x* and *y*, $\frac{\partial f}{\partial u}$ and $\frac{\partial f}{\partial v}$, are:

diff(f(x,y),u);
diff(f(x,y),v);

Implicit differentiation

Functions defined implicitly can be derived using the **implicitdiff** command. For example, suppose we have a surface G, where z is defined implicitly as a function of x and y.

G := x^2-x*y*z-x^2*y^2*z^3 = 0;

Then the partial derivative of *z* with respect to *x*, $\frac{\partial z}{\partial x}$ and with respect to *y*, $\frac{\partial z}{\partial y}$, are given by

implicitdiff(G,z(x,y),x);
implicitdiff(G,z(x,y),y);

For those involving a Jacobian, you can use the command from the **VectorCalculus** package.

```
with(VectorCalculus):
BasisFormat(false):
```

For example, $\frac{dy}{dx} = -\frac{\frac{\partial(f,g)}{\partial(x,z)}}{\frac{\partial(f,g)}{\partial(y,z)}}$ for the curve given by $\begin{cases} x^2+y^2+z^2=2 \\ z-x^2-y^2=0 \end{cases}$ at $\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2},1\right)$ would be:

```
f := x^2+y^2+z^2-2;
g := z - x^2 – y^2;
M1, d1 := Jacobian([f,g],[x,z],'determinant');
M2, d2 := Jacobian([f,g],[y,z],'determinant');
subs(x=sqrt(2)/2, y=sqrt(2)/2, z=1, -d1/d2);
```

Gradients

Maple has a command in the **VectorCalculus** parckage for gradients. For example:

```
f := (x,y,z)->cos(x)*sin(y)-z;
Gradient(f(x,y,z),[x, y, z]);
```

Max and Mins

We can use Maple to help us find local max and mins. Consider $f(x,y)=12xy-4x^2y-3xy^2$.

```
f := (x,y)-> 12*x*y-4*x^2*y-3*x*y^2;
```

We can find the critical points by solving $\frac{\partial f}{\partial x}=0$ and $\frac{\partial f}{\partial y}=0$ using the **solve** command.

```
SOL:=solve({D[1](f)(x,y) = 0, D[2](f)(x,y) = 0},{x, y});
```

To find out if we have a max, min or saddle point we need to find the determinant of the Hessian matrix, i.e $f_{xx}f_{yy}-f_{xy}^2$. (Note, you need the **LinearAlgebra** package)

```
with(LinearAlgebra)
H:=Hessian(f(x,y),[x, y]);
DET:=Determinant(H);
```

All you have to do now is plug in the critical points in this determinant and in f_{xx} if necessary.

```
eval(DET,SOL[1]);
eval(DET,SOL[2]);
eval(DET,SOL[3]);
eval(DET,SOL[4]);
```

Here SOL[i] stands for the i[th] critical point, found earlier. Since the only positive answer is found at SOL[4], then we evaluate f_{xx} and f at this point.

```
D[1,1](f)(1,4/3);
f(1,4/3);
```

Thus we have saddle points at (0,0), (3,0) and (0,4) and a minimum at (1,4/3) of 16/3.

Absolute extrema

If we want to find the absolute max or min of $f(x,y)=x^2+y^2$ on the interval [-5,5], [-5,5]:

```
f := x^2+y^2;
maximize(f,{x, y},{x = -5 .. 5, y = -5 .. 5});
minimize(f,{x, y},{x = -5 .. 5, y = -5 .. 5});
```

Note, this does not tell us where the max or mins are. Also, the boundary is a square. This does not work if we have a boundary other than a square.