MATHEMATICS 201-BNK-05
Advanced Calculus
Martin Huard
Winter 2011

Assignment #4

This assignment is due ***Wednesday April 27, 2011*** at the beginning of the class.
Complete solutions with exact answers are expected, presented in a neat and legible manner.

Question 1 (8 points)

If *f* is continuous, show that $\int_0^t \int_0^y \int_0^z f(x)\,dx\,dz\,dy = \frac{1}{2}\int_0^t (x-t)^2 f(x)\,dx$.

Question 2 (8 points)

Find the center of mass for the solid that is outside the cylinder $x^2 + y^2 = 3$ and inside the cylinder $x^2 + y^2 = 2x$, bounded above and below by the planes $x - z = 0$ and $x + z = 0$, and with a density given by $\delta(x,y,z) = \dfrac{1}{\sqrt{x^2+y^2}}$.

Question 3 (8 points)

Consider the ellipsoid $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$.

a) Find the volume of the ellipsoid using the following change of variables.

$$x = ar\sin\phi\cos\theta$$
$$y = br\sin\phi\sin\theta$$
$$z = cr\cos\phi$$

b) Earth is not a perfect sphere; rotation has resulted in fattening at the poles So the shape can be approximated by an ellipsoid with $a = b = 6378$ km and $c = 6356$ km. Use the answer in (b) to estimate the volume of the earth.

Question 4 (8 points)

Solve the following differential equations.

$$x\frac{dy}{dx} + y = x\sqrt{y}\arctan\sqrt{x}$$

Question 5 (9 points)

Consider a differential equation of the form $yf(xy)\,dx + xg(xy)\,dy = 0$ where $f(xy) \neq g(xy)$.

a) Show that multiplying the above equation by following the integrating factor yields an exact differential equation.

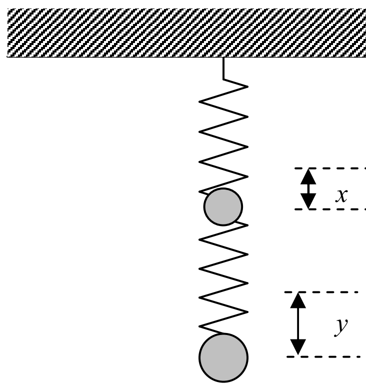
$$\frac{1}{xy\left(f(xy) - g(xy)\right)}$$

b) Use (a) to solve the following differential equation.

$$y(2xy+1)\,dx + x\left(1+2xy - x^3y^3\right)dy = 0 \qquad y(2) = 1$$

Question 6 (9 points)

Suppose that two objects of mass m_1 and m_2 are is attached to elastic springs suspended from the ceiling.



If x and y denote the displacement of the masses m_1 and m_2 from their equilibrium positions, then Hooke's law and Newton's Second law of motion gives the following system of differential equations

$$m_1 \frac{d^2x}{dt^2} = k_2(y - x) - k_1 x$$
$$m_2 \frac{d^2y}{dt^2} = -k_2(y - x)$$

where friction is negligible. Suppose that $m_1 = 2$ kg, $m_2 = 1$ kg, $k_1 = 4$ kg/m and $k_2 = 2$ kg/m. Also, suppose that $x(0) = 2$, $x'(0) = 0$, $y(0) = 3$ and $y'(0) = 0$.

a) Combine the two differential equations together to obtain the following differential equation in terms of x only. *Hint*: *Differentiate the first equation twice to get a new differential equation of degree 4. Then substitute the y's using the two equations given above.*

$$\frac{d^4x}{dt^4} + 5\frac{d^2x}{dt^2} + 4x = 0$$

b) Solve the above differential equation to a general solution for $x(t)$.

c) Use the answer found in (b) to find a general solution for $y(t)$. *Hint: Replace the solution $x(t)$ in the first equation given above.*

d) Find the particular solutions for $x(t)$ and $y(t)$ using the given initial conditions.