MATHEMATICS 201-BNK-05
Advanced Calculus
Martin Huard
Winter 2011

Assignment #3

This assignment is due ***Wednesday March 30, 2011*** at the beginning of the class.
Complete solutions with exact answers are expected, presented in a neat and legible manner.

For questions involving Maple, a print-out of your work is expected, where your name is written in the Worksheet, each question is clearly labeled, and the answers are clearly presented. Also, you must copy your file in my "TEST" subfolder (W:\Tests\mhuard\Advanced Calculus \Assignment 3), where your name should be included in the name of the file (for example: Assignment 3 – Your Name).

Question 1 (10 points)

Find the absolute maximum and minimum values for $f(x,y)=x^3-xy+2y$ over the region R bounded by $y=x^2-5x$ and $y=x+7$.

Question 2 (9 points)

A moat of width a is bounded on each side by a wall of height b. Soldiers plan to bridge this moat by scaling a ladder placed across the nearer wall and anchored at the ground with a boulder, and with the upper end directly above the far wall on the opposite side of the moat. They obviously wish to know the minimal length L of the ladder that is needed.

a) Let the ends of the ladder be at the points $(x,0)$ and $(0,y)$. Show that the ladder must then satisfy the constraint $\dfrac{a}{x}+\dfrac{b}{y}=1$.

b) Minimize the square of the length of the ladder subject to the above constraint to find L.

Question 3 (7 points)

Evaluate the following.

$$\int_0^9\int_{3-\sqrt{9-y}}^{\sqrt{y}}\frac{y}{\sqrt{y^2-x^4}}\,dxdy$$

Question 4 (7 points)

Evaluate the following improper integral, if it converges.

$$\int_{-\infty}^{\infty}\int_0^{\infty}\frac{1}{\left(3+x^2+y^2\right)^{\frac{3}{2}}}\,dydx$$

Question 5 (9 points)

Consider the region R bounded by the leaf $r = 2\cos 2\theta$ (in polar coordinates) that cuts the positive y-axis.

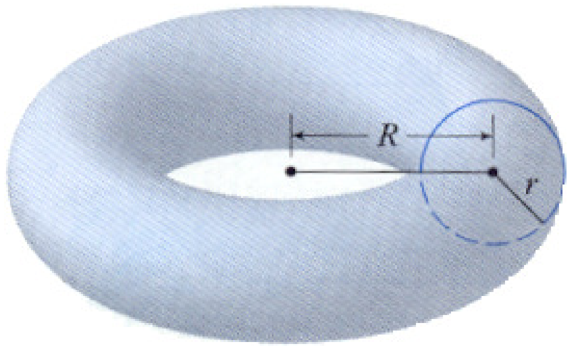
a) Find the centroid $(\bar{x}, \bar{y})$.

b) Use Maple to sketch a graph of the leaf.

c) Verify your answer in (a) with Maple.

Question 6 (8 points)

The **torus** is the donut-shaped solid obtained by revolving a circle of radius r (in the xz-plane) around the z-axis at a distance R from the z-axis.



It can be shown that the torus can be expressed parametrically as

$$x = (R + r\cos v)\cos u$$

$$y = (R + r\cos v)\sin u$$

$$z = r\sin v$$

where $0 \le u \le 2\pi$ and $0 \le v \le 2\pi$.

a) Find the surface area of the torus.

b) Use Maple to sketch the graph of the torus when $R = 3$ and $r = 1$ (with scaling constrained, not to get a deformed view of the torus).

c) Use Maple to verify your answer in (a).