MATHEMATICS 201-BNK-05
Advanced Calculus
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Winter 2011

Assignment #1

This assignment is due ***Tuesday February 8, 2011*** at the beginning of the class.
Complete solutions with exact answers are expected, presented in a neat and legible manner.

For questions involving Maple, a print-out of your work is expected, where your name is written in the Worksheet, each question is clearly labeled, and the answers are clearly presented. Also, you must copy your file in my "TEST" subfolder (W:\Tests\mhuard\Advanced Calculus \Assignment 1), where your name should be included in the name of the file (for example: Assignment 1 – Your Name).

Question 1 (5 points)

Prove that $\lim\limits_{x\to 2^-} \dfrac{1}{\sqrt{x^2+5}-3} = -\infty$.

Question 2 (5 points)

Prove that $\lim\limits_{x\to -\infty} \left(1-x^3\right) = 4$ is false.

Question 3 (5 points)

Prove that the function $f\left(x\right) = \dfrac{x}{x^2+4}$ is continuous at $x = -\frac{1}{2}$.

Question 4 (5 points)

Prove that the function $f\left(x\right) = \begin{cases} x^2+1 & x<1 \\ 3-\frac{1}{2}x & x\geq 1 \end{cases}$ is not continuous at $x=1$.

Question 5 (5 points)

A sequence $\left\{a_n\right\}_{n=0}^{\infty}$ converges to L if $\lim\limits_{n\to\infty} a_n = L$. Give a formal definition for the convergence of a sequence. Note that n is always a nonnegative integer.
Use your definition to prove that the sequence $\left\{\frac{2n}{n+1}\right\}_{n=0}^{\infty}$ converges. Note: you must find the value to which it converges first.

Question 5 (25 points)

Consider the vector function $\vec{r}(t)=\left(\int_0^t \cos\left(\tfrac{\pi}{2}\theta^2\right)d\theta, \int_0^t \sin\left(\tfrac{\pi}{2}\theta^2\right)d\theta, t\right)$. (The two functions in the *x* and *y* coordinate are called **Fresnel functions**. This parameterization is used in the design of transition curves in highways and loops in roller coasters, though the *z* component differs.)

a) Reparameterize the curve with respect to arc length measured from the point where $t=0$ in the direction of increasing *t*.

b) Find the curvature κ.

c) Find the torsion τ (see question 14 in exercise sheet IV).

d) Find the unit tangent vector, unit normal vector and unit binormal vector for the curve at $t=1$.

e) With Maple, find the curvature and torsion for the curve. Compare with your answer in (b) and (c).
Note: For Maple to simplify the expressions adequately, it has to assume that *t* is a real number. Hence, at the beginning, use the command **assume(t:: real);** Maple will use *t*~ instead of *t* after, to indicate that there is an assumption on *t*.

f) Verify your calculations in (d) with Maple.

g) With Maple, plot, on the same graph, the curve give by $\vec{r}(t)$ along with the three vectors found in (d), where the major features of the graph are clearly seen.
Note: Use Maple to give you a numerical approximation for $\vec{r}(1)$.