MATHEMATICS 201-BNJ-05
Topics in Mathematics
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Semester Review

1. Find the sequence $\langle a_n \rangle_{n=0}^{\infty}$
   a) $\langle a_n \rangle$ is arithmetic with $a_3 = -2$ and $a_7 = 5$
   b) $\langle a_n \rangle$ is geometric with $a_5 = 20$ and $a_7 = 125$
   c) $a_n = \frac{2}{3}, \frac{-5}{9}, \frac{8}{27}, \frac{-11}{81}, \frac{14}{243}, \ldots$
   d) $\langle a_n \rangle$ is arithmetic with $a_6 = 1 - i$ and $a_9 = 4 + 2i$
   e) $\langle a_n \rangle$ is geometric with $a_5 = 3 + 2i$ and $a_6 = 5 - i$

2. A rich man promises to give you \$1000 on June 1st, 2006. Each day thereafter he will give you $\frac{9}{10}$ of what he gave on the previous day. What is the first date on which the amount you receive is less than 1¢ (when the money received, rounded to the nearest cent, is less than 1)? How much have you received when this happens?

3. Determine if the following sequences converge. Give the exact limit of any convergent sequence. Identify all attracting, repelling and semi-attracting fixed points. For each attracting fixed point, find an interval so that if the initial value is in the interval, the sequence converges to the fixed point.
   a) $a_{n+1} = 3a_n - 2 \qquad a_0 = 2$
   b) $a_{n+1} = 3a_n - a_n^2 + 3 \qquad a_0 = 2$
   c) $a_{n+1} = \frac{1}{4} - a_n^2 \qquad a_0 = 1$
   d) $a_{n+1} = a_n^3 - 2a_n^2 + 2 \qquad a_0 = \frac{1}{2}$
   e) $a_{n+1} = a_n^3 - a_n^2 + 1 \qquad a_0 = \frac{1}{2}$

4. Prove the following claims using the axiom of mathematical induction.
   a) If $a_0 = 5$ and $a_{n+1} = 3a_n + 2n - 4$ then $\langle a_n \rangle = \left\langle \frac{7}{2} 3^n + \frac{3}{2} - n \right\rangle_{n=0}^{\infty}$
   b) If $b_0 = 5$, $b_1 = 4$ and $b_{n+2} = 3b_{n+1} + 10b_n$ then $\langle b_n \rangle = \left\langle 2 \cdot 5^n + 3(-2)^n \right\rangle_{n=0}^{\infty}$
   c) $n^3 + 2n$ is divisible by 3
   d) $\sum_{i=1}^{n} i(i+1) = \frac{1}{3} n(n+1)(n+2)$

5. Give the following sequences in closed form.
   a) $a_0 = 5 \qquad a_{n+1} = 3 - \frac{1}{2} a_n$
   b) $b_0 = 3 \qquad b_{n+1} = b_n + 3n - 5$
   c) $a_0 = 3$, $a_1 = 4 \qquad a_{n+2} = 2a_{n+1} + 8a_n$
   d) $b_0 = 1$, $b_1 = 4 \qquad a_{n+2} = 4a_{n+1} - 13a_n + 20$
   e) $c_0 = -2$, $c_1 = 10 \qquad c_{n+2} = 10c_{n+1} - 25c_n + 16$

6. Use basic sigma properties to show $\sum_{i=2}^{n+1}\left(\left(i-1\right)^2 - 2i + 6\right) = \sum_{i=0}^{n}\left(i^2+1\right) - \left(n-1\right)^2$ without expanding.

7. Evaluate $\sum_{i=1}^{3}\sum_{j=-26}^{27} i\sqrt[3]{j}$.

8. Find the sum of the first n terms of $\left\langle 2n - 5^n + 3 \right\rangle_{n=0}^{\infty}$.

9. Find the exact value of the following sums.
   a) $\sum_{n=2}^{20}\left(3n-5\right)$
   b) $\sum_{n=0}^{6}\frac{4\cdot 5^{n+1}}{3\cdot 2^{3n-1}}$
   c) $\sum_{n=0}^{10}\left(2+3i-4n+2ni\right)$
   d) $\sum_{n=0}^{6}\left(1+2i\right)\left(1+i\right)^n$

10. Write the infinite decimal expansion of $3.1\overline{415}$ as quotient of two integers.

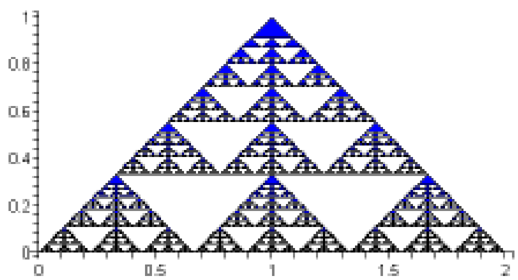
11. Find the planar transformation F that maps triangle ABC with $A = \left(2,-1\right)$, $B = \left(2,3\right)$ and $C = \left(4,3\right)$ to triangle $A'B'C'$ where $A' = f\left(A\right) = A$, $B' = f\left(B\right) = C$ and $C' = f\left(C\right) = \left(2,4\right)$.

12. Find the planar transformation F that maps triangle ABC with $A = \left(-2,1\right)$, $B = \left(-5,1\right)$ and $C = \left(-2,6\right)$ to triangle $A'B'C'$ where $A' = f\left(A\right) = \left(2,4\right)$, $B' = f\left(B\right) = \left(2,-2\right)$ and $C' = f\left(C\right) = \left(12,4\right)$.

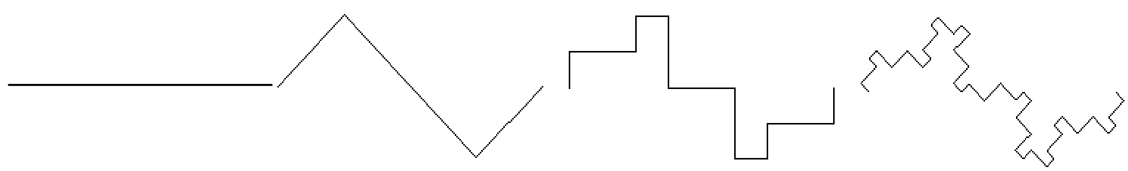
13. Find the image of the line $y = 4x + 3$ under the planar transformation $F = D_2 \circ M_x \circ T_{(-3,1)}$.

14. If $\langle (x_n, y_n) \rangle$ is defined recursively by $(x_0, y_0) = (4,1)$ and $(x_{n+1}, y_{n+1}) = F(x_n, y_n)$, then find (x_n, y_n) in closed form.
   a) $F = T_{(2,-1)} \circ D_{\frac{2}{3}}$
   b) $F(x,y) = (3x - y, 4x + 7y)$
   c) $F(x,y) = (3x - 5y + 1, -3x + y + 2)$
   d) $F = T_{(1,-1)} \circ D_{\frac{1}{2}} \circ R_{\frac{\pi}{3}}$

15. Consider the following fractal A given below.
   a) Give a recursive definition for this fractal.
   b) How would you find the dimension?
   c) How could make a sequence of points (not sets) in the plane having A as an attractor?



16. Consider the following fractal A, constructed as shown below.
   a) Give a recursive definition for this fractal.
   b) Find the dimension.
   c) How could make a sequence of points (not sets) in the plane having A as an attractor?



17. Sketch the graphs of the polar equations, and transform the equations into Cartesian form.
   a) $r = 1 - \sin\theta$
   b) $r = \frac{3}{2 - 2\cos\theta}$

18. Give the following numbers in standard form.
   a) $\dfrac{2-i}{i(3+4i)}$
   b) $\dfrac{(1-i)(2+i)}{(1+i)(2-i)}$
   c) $\sqrt[7]{-2-2\sqrt{3}i}$
   d) $\sqrt{e^{2-2i}}$
   e) $\arccos\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$
   f) $\operatorname{arctanh}\left(2 + \sqrt{3}i\right)$

19. Give the following numbers in trigonometric and in exponential form.

a) $\left(2+2i\right)^7$ b) $\dfrac{\left(-2+2\sqrt{3}i\right)^4\left(-4-4i\right)^2}{1-i}$

20. Find all 8th roots of $-i$ and plot them in the complex plane.

21. Solve the equation $e^z=-\sqrt{3}+i$. Give all solutions.

22. Prove that $\arccos z=-i\ln\left(z+\sqrt{z^2-1}\right)$.

23. Prove that $\operatorname{arcsec} z=-i\ln\left(\dfrac{1}{z}+\sqrt{\dfrac{1}{z^2}-1}\right)$. Use this result to evaluate $\operatorname{arcsec}\left(i\right)$.

24. Describe the function $f\left(z\right)=\left(1-i\right)\left(z+2-3i\right)$ as a composition of simple planar transformations. Then give the image of the circle $C=\left\{z\mid\left|z-1\right|=1\right\}$.

25. Find $\left\langle z_n\right\rangle$ in closed form and then find z_9.

a) $z_0=3-4i$ $z_{n+1}=\left(-3+3i\right)z_n+5-10i$

b) $z_0=3-2i$ $z_{n+1}=T_{2\sqrt{3}+3i}\circ D_4\circ R_{120^\circ}\left(z_n\right)$

26. Find polynomial q and r so that $p=qd+r$ and $\deg\left(r\right)<\deg\left(d\right)$.

a) $p\left(x\right)=2x^5-6x^4+6x^3+14x^2-19x+9$ $d\left(x\right)=x^2-3x+5$

b) $p\left(z\right)=\left(7-i\right)z^5+\left(13+i\right)z^4+\left(8+6i\right)z^3+\left(-3-13i\right)z^2-8z+4+19i$

$d\left(z\right)=\left(3+i\right)z^2-2$

c) $p\left(x\right)=3x^4+\left(3-6\sqrt{2}\right)x^3-2\sqrt{2}x^2+\left(9-2\sqrt{2}\right)x+\sqrt{2}+1$ $d\left(x\right)=x-2\sqrt{2}+1$

27. Factor *f* completely over (i) $\mathbb{Q}$ (ii) $\mathbb{R}$ and (iii) $\mathbb{C}$. (Give the prime factorization)

a) $f\left(x\right)=x^4-4x^3+3x^2+14x+26$ $3-2i$ is a zero of *f*

b) $f\left(z\right)=z^3+\left(6-4i\right)z^2+\left(-2-i\right)z+1+47i$ $z-2+i$ is a factor of *f*

c) $f\left(x\right)=x^6-2x^5+10x^4-16x^3+32x^2-32x+32$ $2i$ is a zero of multiplicity 2

d) $f\left(x\right)=x^4-\frac{25}{6}x^3+\frac{3}{2}x^2+\frac{1}{2}x-\frac{1}{6}$

e) $f\left(x\right)=x^5-4x^4+x^3+10x^2-4x-8$

f) $f\left(x\right)=5x^4+\frac{13}{2}x^3-29x^2-\frac{19}{2}x+3$

g) $f\left(x\right)=x^6+3x^5-6x^3-19x^2-45x-30$

28. Find a zero to two-figure accuracy for $x^3 - 7$ using the bisection method.

29. Find a zero to six-figure accuracy for $x^3 + 4x^2 - 2$, then use this zero to find all the other zeros (to five-figure accuracy).

30. A committee with four members is to be chosen among 7 women and 10 men. What is the probability that the committee will have more men than women?

31. Two urns, A and B, have green and blue marbles. Urn A has 2 green marbles and 4 blue marbles, while urn B has 4 green marbles and 1 blue marble. A die is rolled. If a 1 or 2 comes up, 2 marbles are chosen from urn A, and if a 3, 4, 5 or 6 is chosen, 2 marbles are chosen from urn B.
    a) What is the probability of having two green marbles?
    b) What is the probability of having at least one green marble?
    c) If two green marbles were chosen, what is the probability they came from urn A?

32. Find the coefficient of x^4 in the expression $\left(x - \frac{2}{\sqrt{x}}\right)^{10}$

33. How many five digit numbers are there (a number cannot start with zero),
    a) if there are no repetitions
    b) if the numbers start with a 2 and end with a 4 (without repetitions)
    c) if the numbers don't have any fives (without repetitions)
    d) if the number is divisible by 5 (without repetitions)
    e) if the first digit is even and the last is odd?

34. In how many ways can 14 men be placed into 6 team where the first team has 3 members, the second team has 2 members, the third team has 3 members, and the fourth, fifth, and sixth teams each have 2 members?

35. An insurance company estimates that people fall into two categories, the ones that are inclined to have accidents (I) and the ones that aren't (N). Statistics show that an individual in I has a probability of 0.4 to have an accident in the space of a year, and a probability of 0.2 for those who are in N. If 30% of the population is in I, what is the probability that
    a) A newly insured person has an accident during the next year.
    b) A newly insured person is in the category I if he had an accident during the first year.

36. Teams A and B are in the finals. (The first team that wins two games wins the tournament. No ties allowed). If the probability that team A wins a game is 40% (according to experts), what is the probability that
    a) team A wins the tournament
    b) team B wins at least one game
    c) team A wins the tournament and team B wins at least one game
    d) team A wins the tournament knowing that team B will win at least one game.

37. At an iron foundry, it has been established that the sand used for molding iron casting is too wet 5% of the time and too dry 3% of the time. Also, defective castings occur 1% of the time when the sand has the correct amount of moisture, 7% of the time when the sand is too dry, and 30% of the time when the sand is too wet. Suppose a casting is selected at random and found to be defective, what is the probability the sand was too wet?

38. A group of 25 freshman mathematics students at a large university were surveyed. The results showed that 17 of the students read the *Mathematics Magazine*, 9 *Scientific American* and 5 both. A person from the group of freshman is selected at random,
    a) Find the probability that the person reads the *Mathematics Magazine* or *Scientific American*.
    b) Find the probability that the person reads neither of these magazines.
    c) Find the probability that the person reads the *Mathematics Magazine* given that he reads *Scientific American*.
    d) Find the probability that the person does not read the *Mathematics Magazine* given that he does not read *Scientific American*.

39. The probability that a student applying for engineering at university is accepted is 0.5, the probability of being accepted in law is 0.4 and the probability of being accepted in engineering, law or both is 0.7. What is the probability that a person will be accepted
    a) in both?
    b) in law but not in engineering?

40. Studies have shown that if a person had a car accident one year, the probability they will have another one the next year is 0.5, while if they didn't have any one year, then the probability they will not have one the next is 0.9. If Paul had an accident this year,
    a) what is the probability that he will have a car accident n years from now?
    b) In the long run, what is the proportion of years for which Paul will have a car accident?

41. The probability a student will be late one class is 0.3 if he was late the previous class, and 0.05 if he wasn't late the previous class. Suppose Leonard is late for class today.
    a) What is the probability that he will be late for class n classes from now?
    b) In the long run, what is the proportion of classes for which Leonard will be late?

Answers

1. a) $\langle a_n\rangle = \left\langle \frac{7}{4}n - \frac{29}{4}\right\rangle_{n=0}^{\infty}$ b) $\langle a_n\rangle = \left\langle \frac{128}{625}\left(\frac{5}{2}\right)^n\right\rangle_{n=0}^{\infty}$ c) $\left\langle \frac{(3n+2)(-1)^n}{3^{n+1}}\right\rangle_{n=0}^{\infty}$

   d) $\langle a_n\rangle = \left\langle -5-7i+(1+i)n\right\rangle_{n=0}^{\infty}$ e) $\langle a_n\rangle = \left\langle \left(\frac{-1}{8}-\frac{5}{8}i\right)(1-i)^n\right\rangle_{n=0}^{\infty}$

2. $\langle a_n\rangle = \left\langle 1000\left(\frac{9}{10}\right)^n\right\rangle_{n=0}^{\infty}$ If $n \ge 116$, then $a_n < 0.005$. Thus 116 days later. \$9999.95

3. a) Diverges, FP: $x=1$ is repelling.
   b) Diverges, FP: $x=-1$ and $x=3$ are repelling.
   c) $\langle a_n\rangle \to \frac{-1+\sqrt{2}}{2}$, FP: $x=\frac{-1-\sqrt{2}}{2}$ is repelling, $x=\frac{-1+\sqrt{2}}{2}$ is attracting, conv. for $a_0 \in \left(\frac{-1-\sqrt{2}}{2}, \frac{1+\sqrt{2}}{2}\right)$
   d) $\langle a_n\rangle \to 1$, FP: $x=-1,2$ are repelling, $x=1$ is attracting, conv. for $a_0 \in (-1,0)\cup(0,2)$
   e) $\langle a_n\rangle \to 1$, FP: $x=-1$ is repelling, $x=1$ is semi-attracting, conv. for $a_0 = (-1,0)\cup(0,1)$

5. a) $\langle a_n\rangle = \left\langle 3\left(\frac{-1}{2}\right)^n + 2\right\rangle_{n=0}^{\infty}$ b) $\langle b_n\rangle = \left\langle \frac{3}{2}n^2 - \frac{13}{2}n + 3\right\rangle$ c) $\langle a_n\rangle = \left\langle \frac{5}{3}4^n + \frac{4}{3}(-2)^n\right\rangle_{n=0}^{\infty}$

   d) $\langle b_n\rangle = \left\langle \left(\frac{-1}{2}-\frac{2}{3}i\right)(2+3i)^n + \left(\frac{-1}{2}+\frac{2}{3}i\right)(2-3i)^n + 2\right\rangle_{n=0}^{\infty}$ e) $\langle c_n\rangle = \left\langle \left(-3+\frac{24}{5}n\right)5^n + 1\right\rangle_{n=0}^{\infty}$

7. 18

8. $\frac{-1}{4}5^n + n^2 + 2n + \frac{1}{4}$

9. a) 532 b) $\frac{3365045}{93304}$ c) $-198+143i$ d) $6-23i$

10. $\frac{15692}{4995}$

11. $F(x,y) = \left(-x+\frac{1}{2}y+\frac{9}{2}, x+y-2\right)$

12. $F(x,y) = (2y, 2x+8)$

13. $y=-4x-32$

14. a) $\langle (x_n,y_n)\rangle = \left\langle \left(-2\left(\frac{2}{3}\right)^n + 6, 4\left(\frac{2}{3}\right)^n - 3\right)\right\rangle_{n=0}^{\infty}$ b) $\langle (x_n,y_n)\rangle = \left\langle \left(4-\frac{9}{5}n\right)5^n, \left(1+\frac{18}{5}n\right)5^n\right\rangle_{n=0}^{\infty}$

    c) $\langle (x_n,y_n)\rangle = \left\langle \left(\frac{19}{12}(-2)^n + \frac{7}{4}6^n + \frac{2}{3}, \frac{19}{12}(-2)^n - \frac{21}{20}6^n + \frac{7}{15}\right)\right\rangle_{n=0}^{\infty}$

    d) $\langle (x_n,y_n)\rangle = \left\langle \left(\left(\frac{9-\sqrt{3}}{3}\cos\frac{n\pi}{3} - \frac{6-\sqrt{3}}{3}\sin\frac{n\pi}{3}\right)\left(\frac{1}{2}\right)^n + \frac{3+\sqrt{3}}{3}, \left(\frac{9-\sqrt{3}}{3}\sin\frac{n\pi}{3} + \frac{6-\sqrt{3}}{3}\cos\frac{n\pi}{3}\right)\left(\frac{1}{2}\right)^n + \frac{-3+\sqrt{3}}{3}i\right)\right\rangle_{n=0}^{\infty}$

15. a) $f_1(x,y) = D_{\frac{1}{3}}(x,y) = \left(\frac{1}{3}x, \frac{1}{3}y\right)$ $f_2(x,y) = T_{\left(\frac{1}{3},0\right)}\circ D_{\frac{1}{3}}(x,y) = \left(\frac{1}{3}x+\frac{1}{3}, \frac{1}{3}y\right)$

    $f_3(x,y) = T_{\left(\frac{2}{3},0\right)}\circ D_{\frac{1}{3}}(x,y) = \left(\frac{1}{3}x+\frac{2}{3}, \frac{1}{3}y\right)$ $f_4(x,y) = T_{\left(\frac{1}{6},\frac{1}{3}\right)}\circ D_{\frac{2}{3}}(x,y) = \left(\frac{2}{3}x+\frac{1}{6}, \frac{2}{3}y+\frac{1}{3}\right)$

    Thus $A_0 = \{(0,0)\}$ $A_{n+1} = f_1(A_n)\cup f_2(A_n)\cup f_3(A_n)\cup f_4(A_n)$

    b) The dimension is the number D satisfying the equation $\left(\frac{1}{3}\right)^D + \left(\frac{1}{3}\right)^D + \left(\frac{1}{3}\right)^D + \left(\frac{2}{3}\right)^D = 1$

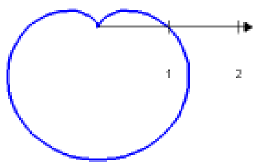
    c) Sequence of points $\langle a_n\rangle$ is defined by $a_0 = (0,0)$, and $a_{n+1} = f_i(a_n)$ where f_i is chosen among the 4 defining functions with probability p_i, where $p_1 = p_2 = p_3 = \frac{1}{7}$ and $p_4 = \frac{4}{7}$.

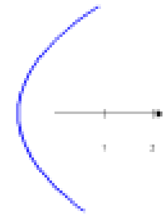
16. a) $f_1(x,y) = \left(\frac{1}{4}x - \frac{1}{4}y, \frac{1}{4}x + \frac{1}{4}y\right)$, $f_2(x,y) = \left(\frac{1}{4}x + \frac{1}{4}y + \frac{1}{4}, -\frac{1}{4}x + \frac{1}{4}y + \frac{1}{4}\right)$,

    $f_3(x,y) = \left(\frac{1}{4}x + \frac{1}{4}y + \frac{1}{2}, -\frac{1}{4}x + \frac{1}{4}y\right)$, $f_4(x,y) = \left(\frac{1}{4}x - \frac{1}{4}y + \frac{3}{4}, \frac{1}{4}x + \frac{1}{4}y - \frac{1}{4}\right)$

    Thus $A_0 = \{(t,0) \mid 0 \le t \le 1\}$ $A_{n+1} = f_1(A_n)\cup f_2(A_n)\cup f_3(A_n)\cup f_4(A_n)$

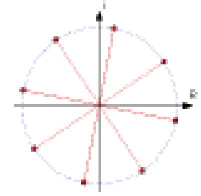
    b) $\frac{4}{3}$ c) Sequence of points $\langle a_n\rangle$ is defined by $a_0 = (0,0)$, and $a_{n+1} = f_i(a_n)$ where f_i is chosen among the 4 defining functions with probability $p_i = \frac{1}{4}$.

17. a) 

$\left(x^2+y^2+y\right)^2 = x^2+y^2$ b) $x=\frac{1}{3}y^2-\frac{3}{4}$ 

18. a) $\frac{-11}{25}-\frac{2}{25}i$ b) $\frac{4}{5}-\frac{3}{5}i$ c) $-\frac{\sqrt[3]{4}}{2}-\frac{\sqrt[3]{4}\sqrt{3}}{2}i$ d) $e\left(\cos(-1)+i\sin(-1)\right)$
e) $\frac{\pi}{4}-i\ln\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{6}}{2}\right)$ f) $\frac{1}{4}\ln 3+\frac{5\pi}{12}i$

19. a) $1024\sqrt{2}\left(\cos\left(\frac{-\pi}{4}\right)+i\sin\left(\frac{-\pi}{4}\right)\right)=1024\sqrt{2}e^{\frac{-\pi}{4}i}$ b) $4096\sqrt{2}\left(\cos\frac{-7\pi}{12}+i\sin\frac{-7\pi}{12}\right)=4096\sqrt{2}e^{\frac{-7\pi}{12}i}$

20. $\cos\left(\frac{-\pi+4\pi k}{16}\right)+i\sin\left(\frac{-\pi+4\pi k}{16}\right)$ $k=0,1,2,3,4,5,6,7$ 

21. $z=\ln 2+\left(\frac{5\pi}{6}+2\pi k\right)i$ $k\in\mathbb{Z}$

23. $\operatorname{arcsec}(i)=\frac{\pi}{2}-i\ln\left(\sqrt{2}-1\right)$

24. $f=T_{-1-5i}\circ D_{\sqrt{2}}\circ R_{\frac{-\pi}{4}}$ $f(C)=\left\{z \mid |z-6i|=\sqrt{2}\right\}$

25. a) $\langle z_n\rangle=\left\langle(1-3i)(-3+3i)^n+2-i\right\rangle_{n=0}^{\infty}$ $z_9=629858+1259711i$
b) $\langle z_n\rangle=\left\langle(3-3i)\left(-2+2\sqrt{3}i\right)^n+i\right\rangle_{n=0}^{\infty}$ $z_9=786432-786431i$

26. a) $q(x)=2x^3-4x+2$ $r(x)=7x-1$
b) $q(z)=(2-i)z^3+(4-i)z^2+4z-5i$ $r(z)=4+9i$
c) $q(x)=3x^3-2\sqrt{2}x+1$ $r(x)=3\sqrt{2}$

27. a) (i) (ii) $\left(x^2-6x+13\right)\left(x^2+2x+2\right)$ (iii) $(x-3+2i)(x-3-2i)(x+1+i)(x+1-i)$
b) only (iii) applies : $(z-2+i)(z+3-2i)(z+5-3i)$
c) (i) (ii) $\left(x^2+4\right)^2\left(x^2-2x+2\right)$ (iii) $(x-2i)^2(x+2i)^2(x-1+i)(x-1-i)$
d) (i) $\left(x-\frac{1}{2}\right)\left(x+\frac{1}{3}\right)\left(x^2-4x+1\right)$ (ii) (iii) $\left(x-\frac{1}{2}\right)\left(x+\frac{1}{3}\right)\left(x-2+\sqrt{3}\right)\left(x-2-\sqrt{3}\right)$
e) (i) (ii) (iii) $(x-2)^3(x+1)^2$ f) (i) (ii) (iii) $5(x-2)\left(x+\frac{1}{2}\right)\left(x-\frac{1}{5}\right)(x+3)$
g) (i) $(x+1)(x+2)\left(x^2+3\right)\left(x^2-5\right)$ (ii) $(x+1)(x+2)\left(x^2+3\right)\left(x-\sqrt{5}\right)\left(x+\sqrt{5}\right)$
(iii) $(x+1)(x+2)\left(x+\sqrt{3}i\right)\left(x-\sqrt{3}i\right)\left(x-\sqrt{5}\right)\left(x+\sqrt{5}\right)$

28. 1.9 29. $-3.86620, -0.789244, 0.655442$ 30. $\frac{15}{34}$

31. a) $\frac{19}{45}$ b) $\frac{13}{15}$ c) $\frac{1}{19}$ 32. 3360

33. a) 27216 b) 336 c) 13440 d) 5712 e) 3720

34. 151 351 200 35. a) 0.26 b) 0.46

36. a) 0.352 b) 0.840 c) 0.192 d) 0.229 37. 0.5703

38. a) $\frac{21}{25}$ b) $\frac{4}{25}$ c) $\frac{5}{9}$ d) $\frac{1}{4}$ 39. a) 0.3 b) 0.1

40. a) $P(A_n)=\frac{5}{6}\left(\frac{2}{5}\right)^n+\frac{1}{6}$ b) Have an accident one in six years

41. a) $P(L_n)=\frac{14}{15}\left(\frac{1}{4}\right)^n+\frac{1}{15}$ c) Late one class out of 15