MATHEMATICS 201-BNJ-05
Topics in Mathematics
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Assignment #4
SOLUTIONS

PART I

Do the following questions from the book.

7.1.14 page 176 (4 points)

$$\begin{array}{r|rrrrr} & 1 & -3+2i & 4-6i & -2+8i & -4i \\ & & -2i & 6i & -8i & 4i \\ \hline -2i & 1 & -3 & 4 & -2 & 0 \end{array}$$

Thus $x^4+(-3+2i)x^3+(4-6i)x^2+(-2+8i)x-4i=(x+2i)(x^3-3x^2+4x-2)$.

Since 1 is a zero for x^3-3x^2+4x-2, then

$$\begin{array}{r|rrrr} & 1 & -3 & 4 & -2 \\ & & 1 & -2 & 2 \\ \hline 1 & 1 & -2 & 2 & 0 \end{array}$$

Hence $x^4+(-3+2i)x^3+(4-6i)x^2+(-2+8i)x-4i=(x+2i)(x-1)(x^2-2x+2)$.

Solving $x^2-2x+2=0$

$$x=\frac{2\pm\sqrt{4-8}}{2}=1\pm i$$

So the solutions are $-2i$, 1, $1+i$ and $1-i$.

7.2.20 page 183 (4 points)

$c = 3i \quad \Rightarrow \quad \frac{c}{3} = i$

$d = -1 - i \quad \Rightarrow \quad \frac{d}{2} = \frac{-1}{2} - \frac{1}{2}i$

Thus

$$\begin{aligned} z &= \left(\tfrac{1}{2} + \tfrac{1}{2}i + \sqrt{\left(\tfrac{-1}{2} - \tfrac{1}{2}i\right)^2 + i^3}\right)^{\frac{1}{3}} \\ &= \left(\tfrac{1}{2} + \tfrac{1}{2}i + \sqrt{\tfrac{1}{2}i - i}\right)^{\frac{1}{3}} \\ &= \left(\tfrac{1}{2} + \tfrac{1}{2}i + \sqrt{-\tfrac{1}{2}i}\right)^{\frac{1}{3}} \\ &= \left(\tfrac{1}{2} + \tfrac{1}{2}i + \sqrt{\tfrac{1}{2}\left(\cos\tfrac{-\pi}{2} + i\sin\tfrac{-\pi}{2}\right)}\right)^{\frac{1}{3}} \\ &= \left(\tfrac{1}{2} + \tfrac{1}{2}i + \tfrac{\sqrt{2}}{2}\left(\cos\tfrac{-\pi}{4} + i\sin\tfrac{-\pi}{4}\right)\right)^{\frac{1}{3}} \\ &= \left(\tfrac{1}{2} + \tfrac{1}{2}i + \tfrac{\sqrt{2}}{2}\left(\cos\tfrac{-\pi}{4} + i\sin\tfrac{-\pi}{4}\right)\right)^{\frac{1}{3}} \\ &= \left(\tfrac{1}{2} + \tfrac{1}{2}i + \tfrac{1}{2} - \tfrac{1}{2}i\right)^{\frac{1}{3}} \\ &= 1^{\frac{1}{3}} \\ &= \left(\cos 0 + i\sin 0\right)^{\frac{1}{3}} \\ &= \cos\tfrac{2\pi k}{3} + i\sin\tfrac{2\pi k}{3} \qquad k = 0,1,2 \\ &= 1,\ \tfrac{-1}{2} + \tfrac{\sqrt{3}}{2}i,\ \tfrac{-1}{2} - \tfrac{\sqrt{3}}{2}i \end{aligned}$$

$$\begin{aligned} k &= \left(-\tfrac{1}{2} - \tfrac{1}{2}i + \sqrt{\left(\tfrac{-1}{2} - \tfrac{1}{2}i\right)^2 + i^3}\right)^{\frac{1}{3}} \\ &= \left(\tfrac{1}{2} + \tfrac{1}{2}i + \sqrt{\tfrac{1}{2}i - i}\right)^{\frac{1}{3}} \\ &= \left(\tfrac{1}{2} + \tfrac{1}{2}i + \sqrt{-\tfrac{1}{2}i}\right)^{\frac{1}{3}} \\ &= \left(-\tfrac{1}{2} - \tfrac{1}{2}i + \tfrac{1}{2} - \tfrac{1}{2}i\right)^{\frac{1}{3}} \\ &= \left(-i\right)^{\frac{1}{3}} \\ &= \left(\cos\tfrac{-\pi}{2} + i\sin\tfrac{-\pi}{2}\right)^{\frac{1}{3}} \\ &= \cos\tfrac{\frac{-\pi}{2} + 2\pi k}{3} + i\sin\tfrac{\frac{-\pi}{2} + 2\pi k}{3} \qquad k = 0,1,2 \\ &= \tfrac{\sqrt{3}}{2} - \tfrac{1}{2}i,\ i,\ -\tfrac{\sqrt{3}}{2} - \tfrac{1}{2}i \end{aligned}$$

Choosing the roots so that $zk = \frac{c}{3} = i$.

Thus, we have

$z = 1 \quad k = i \quad zk = i \qquad x = 1 - i$

$z = \frac{-1}{2} + \frac{\sqrt{3}}{2}i \quad k = \frac{\sqrt{3}}{2} - \frac{1}{2}i \quad zk = i \qquad x = \frac{-1}{2} - \frac{\sqrt{3}}{2} + \left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right)i$

$z = \frac{-1}{2} - \frac{\sqrt{3}}{2}i \quad k = -\frac{\sqrt{3}}{2} - \frac{1}{2}i \quad zk = i \qquad x = \frac{-1}{2} + \frac{\sqrt{3}}{2} + \left(-\frac{\sqrt{3}}{2} + \frac{1}{2}\right)i$

Ergo, the solutions are $1 - i$, $\frac{-1}{2} - \frac{\sqrt{3}}{2} + \left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right)i$ and $\frac{-1}{2} + \frac{\sqrt{3}}{2} + \left(-\frac{\sqrt{3}}{2} + \frac{1}{2}\right)i$

7.3.42 page 191 (6 points)

$$p(x)=\tfrac{1}{2}x^6-\tfrac{1}{3}x^5-\tfrac{11}{6}x^4+\tfrac{2}{3}x^3+\tfrac{1}{6}x^2+x+\tfrac{5}{2}$$
$$=\tfrac{1}{6}\left(3x^6-2x^5-11x^4+4x^3+x^2+6x+15\right)$$

If $\frac{c}{d}$ is a reduced zero of p, then

$c\,|\,15 \quad \Rightarrow \quad x=\pm1,\pm3,\pm5,\pm15$

$d\,|\,3 \quad \Rightarrow \quad x=\pm1,\pm3$

Thus $\frac{c}{d}=\pm\frac{1}{3},\pm1,\pm\frac{5}{3},\pm3,\pm5,\pm15$

$x=\frac{1}{3}$
$$\begin{array}{r|rrrrrrr} & 3 & -2 & -11 & 4 & 1 & 6 & 15 \\ & & -1 & \frac{-1}{3} & \frac{-34}{9} & \frac{2}{27} & \frac{29}{81} & \frac{515}{243} \\ \hline \frac{1}{3} & 3 & -1 & \frac{-34}{3} & \frac{2}{9} & \frac{29}{27} & \frac{515}{81} & \frac{4160}{243} \end{array}$$
$p\left(\frac{1}{3}\right)\neq 0$

$x=1$
$$\begin{array}{r|rrrrrrr} & 3 & -2 & -11 & 4 & 1 & 6 & 15 \\ & & 3 & 1 & -10 & -6 & -5 & 1 \\ \hline 1 & 3 & 1 & -10 & -6 & -5 & 1 & 16 \end{array}$$
$p(1)\neq 0$

$x=\frac{5}{3}$
$$\begin{array}{r|rrrrrrr} & 3 & -2 & -11 & 4 & 1 & 6 & 15 \\ & & 5 & 5 & -10 & -10 & -15 & -15 \\ \hline \frac{5}{3} & 3 & 3 & -6 & -6 & -9 & -9 & 0 \end{array}$$
$p\left(\frac{5}{3}\right)=0$

$$p(x)=\tfrac{1}{6}\left(x-\tfrac{5}{3}\right)\left(3x^5+3x^4-6x^3-6x^2-9x-9\right)$$
$$=\tfrac{1}{2}\left(x-\tfrac{5}{3}\right)\left(x^5+x^4-2x^3-2x^2-3x-3\right)$$

If $\frac{c}{d}$ is a reduced zero of p, then $\frac{c}{d}=\pm1,\pm3$

$$\begin{array}{r|rrrrrr} & 1 & 1 & -2 & -2 & -3 & -3 \\ & & 3 & 12 & 30 & 84 & 243 \\ \hline 3 & 1 & 4 & 10 & 28 & 81 & 240 \end{array}$$

So there are no real zeros greater than 3.

$x=-1$
$$\begin{array}{r|rrrrrr} & 1 & 1 & -2 & -2 & -3 & -3 \\ & & -1 & 0 & 2 & 0 & 3 \\ \hline -1 & 1 & 0 & -2 & 0 & -3 & 0 \end{array}$$
$p(-1)=0$

$$p(x)=\tfrac{1}{6}\left(x-\tfrac{5}{3}\right)\left(3x^5+3x^4-6x^3-6x^2-9x-9\right)$$
$$=\tfrac{1}{2}\left(x-\tfrac{5}{3}\right)(x+1)\left(x^4-2x^2-3\right)$$
$$=\tfrac{1}{2}\left(x-\tfrac{5}{3}\right)(x+1)\left(x^2-3\right)\left(x^2+1\right)$$

a) over $\mathbb{Q}$: $p(x)=\tfrac{1}{2}\left(x-\tfrac{5}{3}\right)(x+1)\left(x^2-3\right)\left(x^2+1\right)$

b) over $\mathbb{R}$: $p(x)=\tfrac{1}{2}\left(x-\tfrac{5}{3}\right)(x+1)\left(x-\sqrt{3}\right)\left(x+\sqrt{3}\right)\left(x^2+1\right)$

c) over $\mathbb{C}$: $p(x)=\tfrac{1}{2}\left(x-\tfrac{5}{3}\right)(x+1)\left(x-\sqrt{3}\right)\left(x+\sqrt{3}\right)(x+i)(x-i)$

7.4.8 page 199 (3 points)

Let $p(x) = x^4 - 2$

$p(1) < 0$

$p(2) > 0$

$x_1 = 1.5$	$p(1.5) = 3.065 > 0$	$\therefore c \in (1, 1.5)$
$x_2 = 1.25$	$p(1.25) = 0.4414 > 0$	$\therefore c \in (1, 1.25)$
$x_3 = 1.125$	$p(1.125) = -0.3982 < 0$	$\therefore c \in (1.125, 1.25)$
$x_4 = 1.1875$	$p(1.1875) = -0.0115 < 0$	$\therefore c \in (1.1875, 1.25)$
$x_5 = 1.21875$	$p(1.21875) = 0.2063 < 0$	$\therefore c \in (1.1875, 1.21875)$

Thus $x \approx 1.2$

7.4.18 page 199 + 22 (for #18) (4 points)

Let $f(x) = x^3 + 3x^2 - 24x - 40$.

We have $f(0) = -40 < 0$ and $f(-2) = 12 > 0$, so by the bisection method we begin with $x_1 = -1$.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_{n+1})} = x_n - \frac{x_n^3 + 3x_n^2 - 24x_n - 40}{3x_n^2 + 6x_n - 24} = \frac{2x_n^3 + 3x_n^2 + 40}{3x_n^2 + 6x_n - 24}$$

$x_2 = -1.518518519$

$x_3 = -1.526840835$

$x_4 = -1.523842524$

$x_5 = -1.523842524$

To six figures, a zero is $x = -1.52384$

22.

	1	3	-24	-40
		-1.52384	-2.24943	40
-1.52384	1	1.47616	-26.24943	0

$f(x) = (x + 1.52384)(x^2 - 1.47616x - 26.24943)$

Solving $x^2 - 1.47616x - 26.24943 = 0$

$$x = \frac{1.47616 \pm \sqrt{1.47616^2 + 4(26.24943)}}{2}$$

Thus the zeros of $x^3 + 3x^2 - 24x - 40$ are -1.5238, -5.9144 and 4.4382.

8.1.44 page 208 (2 points)

$$\frac{17!}{4!5!6!2!}=85765680$$

8.1.52 page 208 (2 points)

$$\binom{15}{4}\binom{11}{3}=(1365)(165)=225225$$

8.1.62 page 209 (3 points)

$$1\binom{6}{2}\binom{4}{2}P(8,2)+7\binom{6}{3}\binom{3}{2}7=7980$$

8.2.12 page 213 (3 points)

$$\left(x^2-\frac{2}{x^2}\right)^8=\sum_{i=0}^{8}\binom{8}{i}\left(x^2\right)^{8-i}\left(\tfrac{-2}{x^2}\right)^i=\sum_{i=0}^{8}\binom{8}{i}(-1)^i\,2^i\,x^{16-4i}$$

Thus the x^4 term will be when $16-4i=4$

$i=3$

Thus the coefficient of the x^4 term is $\binom{8}{3}(-1)^3\,2^3=-448$

For the constant term, we have $16-4i=0$

$i=4$

Thus the constant term is $\binom{8}{4}(-1)^4\,2^4=1120$

9.1.18 page 223 (4 points)

a) $P(2\text{ spoiled})=\frac{\binom{3}{2}}{\binom{12}{2}}=\frac{3}{66}=\frac{1}{22}$

b) $P(\text{neither are spoiled})=\frac{\binom{9}{2}}{\binom{12}{2}}=\frac{36}{66}=\frac{6}{11}$

c) $P(\text{exactly 1 is spoiled})=\frac{\binom{9}{1}\binom{3}{1}}{\binom{12}{2}}=\frac{27}{66}=\frac{9}{22}$

9.1.26 page 224 (2 points)

$$\frac{\binom{20}{12}+\binom{20}{13}+\binom{20}{14}+\binom{20}{15}+\binom{20}{16}+\binom{20}{17}+\binom{20}{18}+\binom{20}{19}+\binom{20}{20}}{2^{20}}=\frac{263950}{1048576}\simeq 0.2517$$

9.1.32 page 224 (3 points)

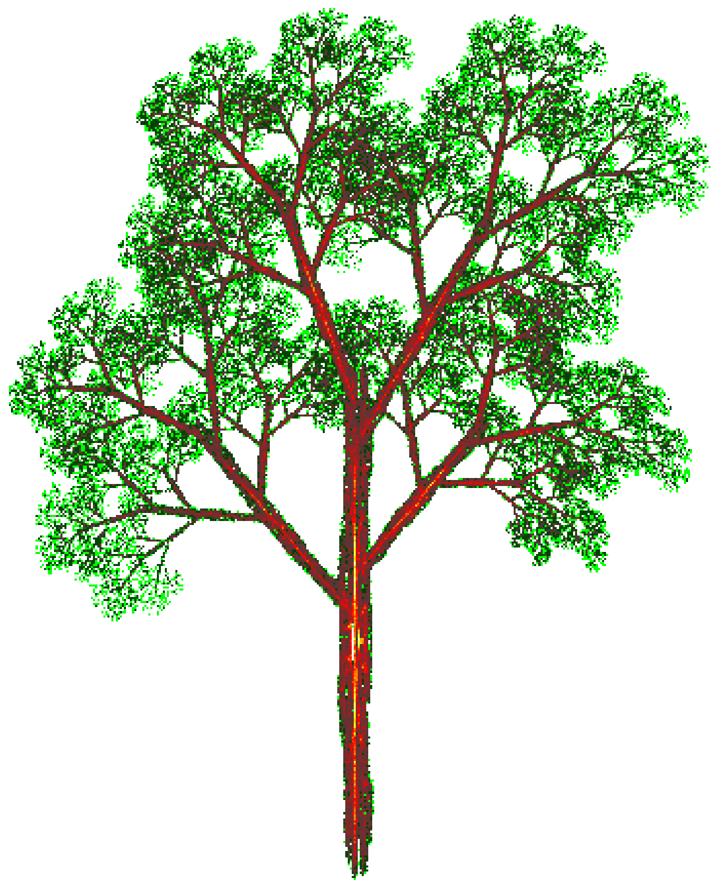
```
Coins := proc( )
local s, i;
s := 0 :
for i from 1 to 5 do
    s := s + rand(0..1)( )
end do:
return(s, tails)
end proc:
```

```
Coins( );
```

4, *tails*

PART II – Due Friday May 1st (10 points)

Consider the following object.



With the help of Maple,

a) Find the defining functions (all are affine)
b) Find the probabilities associated with each function.
c) Sketch an approximation to the fractal.