

## MATHEMATICS 201-BNJ-05

Topics in Mathematics

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Winter 2008

# Assignment #1

This assignment is due *Monday February 2, 2008* at the beginning of the class. Complete solutions with exact answers are expected.

For questions involving Maple, a print-out of your work is expected, where your name is written in the Worksheet, each question is clearly labeled, and the answers are clearly presented. Also, you must copy your file in my “TEST” subfolder (W:\Tests\mhuard\Topics\Assignment 1), where your name should be included in the name of the file (for example: Assignment 1 – Your Name).

## Part I

Do the following questions from the book.

1.2.50 page 11 (3 points)

1.4.12 page 24 along with question 17 (for number 12) (9 points)

1.5.4 page 29 (6 points)

1.7.6 page 43 (2 points)

## Part II

**Question 1** (13 points)

Consider the following number  $\sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots}}}}}$

- Express this as a dynamical system  $\langle a_n \rangle$ .
- Find the fixed point(s) and determine whether it is attracting, repelling, or semi-repelling.
- Use a cobweb diagram to show that the dynamical system found in (a) has a solution that converges to the fixed point for almost any initial value.
- Write a procedure in Maple that will give the  $n^{\text{th}}$  term of the dynamical system  $\langle a_n \rangle$  (in decimal form!) if  $a_1 = 1$ .
- Write a procedure in Maple that will give will list the first  $n$  terms of the dynamical system  $\langle a_n \rangle$  (in decimal form!) if  $a_1 = 1$ .

**Question 2** (9 points)

*This problem is to be done entirely with Maple.*

Consider the dynamical system  $b_{n+1} = 2|b_n - 1|$ .

- Show that if  $b_0 > 2$  or  $b_0 < 0$  then the system diverges. (Show with cobweb diagrams made with Maple.)
- Find a 3-cycle and illustrate with a cobweb diagram.
- How many 3-cycles does the dynamical system have?
- Find a 6-cycle and illustrate with a cobweb diagram.
- Find a  $b_0$  such that the sequence is chaotic. (Show with a cobweb diagram made with Maple).

**Question 3** (4 points)

*This problem is to be done entirely with Maple.*

Consider the dynamical system given by  $a_{n+1} = a_n^2 - a_n + 1$ .

- Find a 2-cycle and illustrate with a cobweb diagram.
- Is the 2-cycle found in (a) is attracting or repulsing?

**Question 4** (4 points)

Write a procedure in Maple that given a nonnegative integer  $n$ , will add all the prime numbers between 1 and  $n$ .