MATHEMATICS 201-510-LW
Business Statistics
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XXV – Inferences about the Population Variance
SOLUTIONS

1. The manufacturer of a certain brand of light bulbs claims that the variance of the lives of these bulbs is 4000 square hours. A consumer agency took a random sample of 25 such bulbs and tested them. The variance of the lives of these bulbs was found to be 4990 square hours. Assume that the lives of all such bulbs are (approximately) normally distributed.
   a) Make the 99% confidence interval for the variance and standard deviation of the lives of all such bulbs.

Step 1 Assumptions: The population is normally distributed

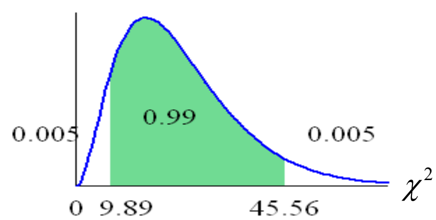
Step 2 a) Test statistic: χ^2 with $df = 25 - 1 = 24$

b) Level of confidence: $1-\alpha = 0.99$ or $\alpha = 0.01$

Step 3 Point estimate: $s^2 = 4990$ hours^2

Step 4 a) $\chi^2_{\left(df,1-\frac{\alpha}{2}\right)} = \chi^2_{(24,0.995)} = 9.89$

$\chi^2_{\left(df,\frac{\alpha}{2}\right)} = \chi^2_{(24,0.005)} = 45.56$



b) $$\frac{(n-1)s^2}{\chi^2_{\left(df,\frac{\alpha}{2}\right)}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{\left(df,1-\frac{\alpha}{2}\right)}}$$

$$\frac{24\cdot 4990}{45.56} < \sigma^2 < \frac{6\cdot 4990}{9.89}$$

$$2628 < \sigma^2 < 12109$$

$$51.3 < \sigma < 110.0$$

Step 5 The 99% confidence interval for the variance of the life of light bulbs is 2628 hours^2 to 12109 hours^2, and for the standard deviation 51.3 hours to 110.0 hours.

b) Test at the 1% significance level if the variance of such bulbs is different from 4000 square hours. Try with both approaches, the classical and the p-value.

Classical Approach

Step 1 Assumptions: The population is normally distributed

Step 2 H_o: $\sigma^2 = 4000$ hours^2

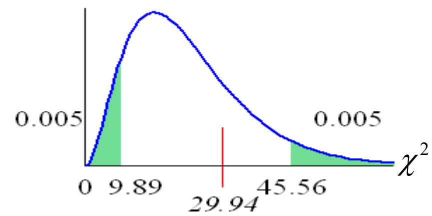
H_A: $\sigma^2 \neq 4000$ hours^2

Step 3 a) Test statistic: χ^2 with $df = 25 - 1 = 24$

b) Two-tailed test with $\alpha = 0.01$

c) $\chi^2_{\left(df,1-\frac{\alpha}{2}\right)} = \chi^2_{(24,0.995)} = 9.89$

$\chi^2_{\left(df,\frac{\alpha}{2}\right)} = \chi^2_{(6,0.005)} = 45.56$



Step 4 $\chi^2 = \dfrac{(n-1)s^2}{\sigma^2} = \dfrac{24 \cdot 4990}{4000} = 29.94$

Step 5 a) χ^2 is not in the critical region

b) Fail to reject H_o.

$\therefore$ There is not sufficient evidence at the 1% level of significance to conclude that the variance of light bulbs is different from 4000 squared hours.

***p*-value approach**

Step 1 Assumptions: The population is normally distributed

Step 2 H_o: $\sigma^2 = 4000$ hours^2

H_A: $\sigma^2 \neq 4000$ hours^2

Step 3 a) Test statistic: χ^2 with $df = 25 - 1 = 24$

b) Two-tailed test with $\alpha = 0.01$

Step 4 $\chi^2 = \dfrac{(n-1)s^2}{\sigma^2} = \dfrac{24 \cdot 4990}{4000} = 29.94$

$2 \cdot 0.100 < p\text{-value} < 2 \cdot 0.250$

$0.200 < p\text{-value} < 0.\ 500$

Step 5 a) $p\text{-value} > 0.200 > \alpha = 0.01$

b) Fail to reject H_o.

$\therefore$ There is not sufficient evidence at the 1% level of significance to conclude that the variance of light bulbs is different from 4000 squared hours.

2. The following are the prices of the same brand of camcorder found at eight stored in Montreal.

$749 815 789 799 732 825 799 769

a) Make the 95% confidence interval for the population standard deviation.

Step 1 Assumptions: The population is normally distributed

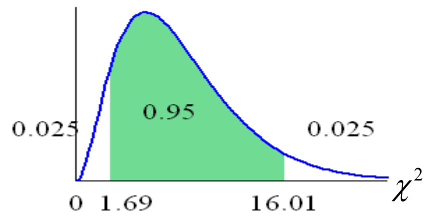
Step 2 a) Test statistic: χ^2 with $df = 8 - 1 = 7$

b) Level of confidence: $1-\alpha = 0.95$ or $\alpha = 0.05$

Step 3 Point estimate: $s^2 = 1038.3$ squared dollars

Step 4 a) $\chi^2_{\left(df,1-\frac{\alpha}{2}\right)} = \chi^2_{(7,0.975)} = 1.69$

$\chi^2_{\left(df,\frac{\alpha}{2}\right)} = \chi^2_{(7,0.025)} = 16.01$



b) $$\frac{(n-1)s^2}{\chi^2_{\left(df,\frac{\alpha}{2}\right)}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{\left(df,1-\frac{\alpha}{2}\right)}}$$

$$\frac{7\cdot 1038.3}{16.01} < \sigma^2 < \frac{7\cdot 1038.3}{1.69}$$

$$454.0 < \sigma^2 < 4300.5$$

$$21.31 < \sigma < 65.58$$

Step 5 The 95% confidence interval for the standard deviation of prices of camcorder in Montreal is \$21.31 to \$65.58.

b) Test at the 5% significance level if the variance of the population is different from 500 square dollars. Use the classical approach. Find the p-value.

Classical Approach

Step 1 Assumptions: The population is normally distributed

Step 2 H_o: $\sigma^2 = 500$ squared dollars

H_A: $\sigma^2 \neq 500$ squared dollars

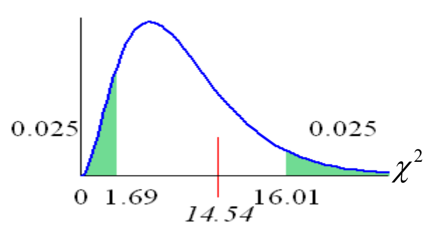
Step 3 a) Test statistic: χ^2 with $df = 8 - 1 = 7$

b) Two-tailed test with $\alpha = 0.05$

c) $\chi^2_{\left(df,1-\frac{\alpha}{2}\right)} = \chi^2_{(7,0.975)} = 1.69$

$\chi^2_{\left(df,\frac{\alpha}{2}\right)} = \chi^2_{(7,0.025)} = 16.01$

Step 4 $$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{7\cdot 1038.3}{500} = 14.54$$



Step 5 a) χ^2 is not in the critical region

b) Fail to reject H_o.

∴ There is not sufficient evidence at the 5% level of significance to conclude that the variance for prices of camcorder in Montreal is different from 500 squared dollars.

***p*-value approach**

Step 1 Assumptions: The population is normally distributed

Step 2 H_o: $\sigma^2 = 500$ squared dollars

H_A: $\sigma^2 \neq 500$ squared dollars

Step 3 a) Test statistic: χ^2 with $df = 8 - 1 = 7$

b) Two-tailed test with α = 0.05

Step 4

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{7 \cdot 1038.3}{500} = 14.54$$

$2 \cdot 0.025 < p\text{-value} < 2 \cdot 0.050$

$0.050 < p\text{-value} < 0.100$

Step 5 a) p-value > α = 0.05

b) Fail to reject H_o.

∴ There is not sufficient evidence at the 5% level of significance to conclude that the variance for prices of camcorder in Montreal is different from 500 squared dollars.

3. A machine currently produces ball bearings whose diameter has a standard deviation of 48.6 microns. The company is considering buying a new machine, and will base her decision on whether the new machine is more accurate, that is, if it has a smaller standard deviation. A random sample of 10 ball bearings made with the new machine is taken, and the diameter (in microns) is measured.

1000.93	990.71	999.11	1000.37	1000.35
1000.16	1007.96	995.13	1001.85	1001.04

a) Make the 90% confidence interval for the variance and standard deviation for the ball bearings produced by the new machine. Assume than the diameter of ball bearings are normally distributed.

Step 1 Assumptions: The population is normally distributed

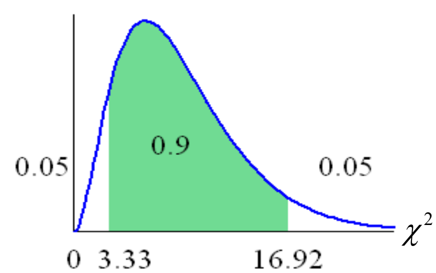
Step 2 a) Test statistic: χ^2 with $df = 10 - 1 = 9$

b) Level of confidence: $1-\alpha = 0.90$ or $\alpha = 0.10$

Step 3 Point estimate: $s^2 = 19.197$ microns^2

Step 4 a) $\chi^2_{(df, 1-\frac{\alpha}{2})} = \chi^2_{(40, 0.95)} = 3.33$

$\chi^2_{(df, \frac{\alpha}{2})} = \chi^2_{(40, 0.05)} = 16.92$



b) $$\frac{(n-1)s^2}{\chi^2_{(df, \frac{\alpha}{2})}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{(df, 1-\frac{\alpha}{2})}}$$

$$\frac{9 \cdot 19.197}{16.92} < \sigma^2 < \frac{9 \cdot 19.197}{3.33}$$

$$10.21 < \sigma^2 < 51.88$$

$$3.20 < \sigma < 7.20$$

Step 5 The 90% confidence interval for the for the ball bearings produced by the new machine is 10.21 microns^2 to 51.88 microns^2, and for the standard deviation is 3.20 microns to 7.20 microns

b) Test at the 10% significance level the claim that the new machine reduces the variance of ball bearing, compared to the old machine. Try with both approaches, the classical and the *p*-value. Assume than the diameter of ball bearings are normally distributed.

Classical Approach

Step 1 Assumptions: The population is normally distributed

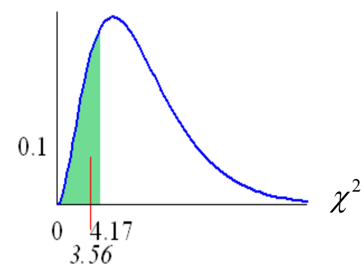
Step 2 H_o: $\sigma^2 = 48.6$ microns^2

H_A: $\sigma^2 < 48.6$ microns^2

Step 3 a) Test statistic: χ^2 with $df = 10 - 1 = 9$

b) Left-tailed test with $\alpha = 0.10$

c) $\chi^2_{(df,1-\alpha)} = \chi^2_{(9,0.90)} = 4.17$



Step 4 $$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{9 \cdot 19.197}{48.6} = 3.56$$

Step 5 a) χ^2 is in the critical region

b) Reject H_o.

$\therefore$ There is sufficient evidence at the 10% level of significance to conclude that the current variance for the diameter of ball bearings is smaller with the new machine.

***p*-value approach**

Step 1 Assumptions: The population is normally distributed

Step 2 H_o: $\sigma^2 = 48.6$ microns^2

H_A: $\sigma^2 < 48.6$ microns^2

Step 3 a) Test statistic: χ^2 with $df = 10 - 1 = 9$

b) Left-tailed test with $\alpha = 0.10$

Step 4 $$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{9 \cdot 19.197}{48.6} = 3.56$$

$1\text{-}0.950 < p\text{-value} < 1\text{-}0.900$

$0.005 < p\text{-value} < 0.10$

Step 5 a) $p\text{-value} < 0.10 < \alpha = 0.10$

b) Reject H_o.

$\therefore$ There is sufficient evidence at the 10% level of significance to conclude that the current variance for the diameter of ball bearings is smaller with the new machine.

4. A machine that puts corn flakes in boxes is adjusted to put an average of 400 grams in each box with a standard deviation of 6 grams. A random sample of 12 boxes gave a sample standard deviation of 9 grams.

   a) Make the 95% confidence interval for the standard deviation for the volume in corn flakes boxes.

Step 1 Assumptions: The population is normally distributed

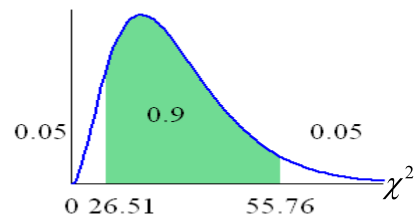
Step 2 a) Test statistic: χ^2 with $df = 12 - 1 = 11$

b) Level of confidence: $1-\alpha = 0.95$ or $\alpha = 0.05$

Step 3 Point estimate: $s^2 = 9^2 = 81$ grams^2

Step 4 a) $\chi^2_{\left(df,1-\frac{\alpha}{2}\right)} = \chi^2_{(11,0.975)} = 3.82$

$\chi^2_{\left(df,\frac{\alpha}{2}\right)} = \chi^2_{(11,0.025)} = 21.92$



b) $$\frac{(n-1)s^2}{\chi^2_{\left(df,\frac{\alpha}{2}\right)}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{\left(df,1-\frac{\alpha}{2}\right)}}$$

$$\frac{11\cdot 81}{21.92} < \sigma^2 < \frac{11\cdot 81}{3.82}$$

$$40.65 < \sigma^2 < 233.25$$

$$6.38 < \sigma < 15.27$$

Step 5 The 95% confidence interval for the standard deviation for the volume in corn flakes boxes is 6.38 grams to 15.27 grams.

   b) Do these data support the claim that the variance has increased and the machine needs to be brought back into adjustment? Use the classical approach. Find the *p*-value.

Classical Approach

Step 1 Assumptions: The population is normally distributed

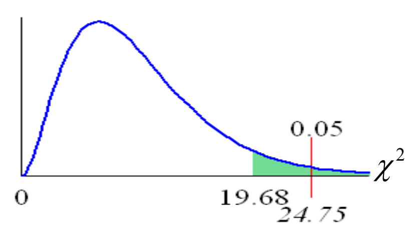
Step 2 H_o: $\sigma^2 = 6^2 = 36$ grams^2

H_A: $\sigma^2 > 36$ grams^2

Step 3 a) Test statistic: χ^2 with $df =$ 12 1 = 11

b) Left-tailed test with $\alpha = 0.05$

c) $\chi^2_{(df,\alpha)} = \chi^2_{(11,0.05)} = 19.68$



Step 4 $$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{11\cdot 81}{36} = 24.75$$

Step 5 a) χ^2 is in the critical region

b) Reject H_o.

∴ There is sufficient evidence at the 5 level of significance to conclude that the machine needs to be adjusted

***p*-value approach**

Step 1 Assumptions: The population is normally distributed

Step 2 H_o: $\sigma^2 = 6^2 = 36$ grams^2

H_A: $\sigma^2 > 36$ grams^2

Step 3 a) Test statistic: χ^2 with df = 12 – 1 = 11

b) Left-tailed test with α = 0.05

Step 4 $\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{11 \cdot 81}{36} = 24.75$

$0.005 < p\text{-value} < 0.01$

Step 5 a) $p\text{-value} < 0.01 < \alpha = 0.05$

b) Reject H_o.

∴ There is sufficient evidence at the 5 level of significance to conclude that the machine needs to be adjusted.