

MATHEMATICS 201-510-LW

Business Statistics

Martin Huard

Fall 2008

XXIV – Chi-Square Goodness of Fit

1. The following table lists the frequency distribution for a sample of 50 absences of college students from classes according to the day of occurrence.

Day of the week	Mon	Tue	Wed	Thu	Fri
Number of absences	14	6	4	10	16

Test at the 5% significance level if the number of absences is equally distributed over all days of the week. Try with both approaches, the classical and the p -value.

2. To check a die for fairness, it is rolled 300 times. If the die is fair, then all of the outcomes should be equally likely. Here are the results of the rolls.

Number on die	1	2	3	4	5	6
Number of rolls	57	43	44	53	46	57

At the 5% level of significance, test the claim that the die is fair. Try with both approaches, the classical and the p -value.

3. A machine currently used by a manufacturer produces units which can be classified as Top Grade, High Grade, Medium Grade and Low Grade. The machine currently produces units of each grade in the proportions of 40%, 30%, 20% and 10% respectively. A new machine, designed to produce the same units, was tested with a random sample of 500 units, where each unit was classified. Here are the results.

	Number of Units
Top Grade	212
High Grade	169
Medium Grade	81
Low Grade	38

At the 10% level of significance, test the claim the two machines produces units of the same quality. Try with both approaches, the classical and the p -value.

4. A pair of dice are rolled 200 times. Here are the sums produced by the 200 rolls.

Sum	2	3	4	5	6	7	8	9	10	11	12
Number of Rolls	4	16	17	16	27	43	25	18	15	16	3

At the 1% significance level, can we conclude that the dice are fair? That is, the sums produced by this pair of dice follow the expected distribution? Try with both approaches, the classical and the p -value.

5. A philosophy instructor tells his students on the first day of class that 40% will pass, 30% will fail, and 30% will withdraw that semester if historical patterns hold true. The class began with 38 students. Seventeen of the students passed the class, 6 failed and the rest dropped. At the 5% level of significance, test the claim that the instructor made on the first day of class. Try with both approaches, the classical and the p -value.
6. A 1970 study showed that of Canadian married-couple families, 42.9% had no children, 18.3% had one child, 18.0% had two children, and 20.8% had three or more children. A recent survey of 500 Canadian married-couple, families revealed that 267 had no children, 87 had one child, 96 had two children, and 50 had three or more children. At the 1% level of significance, test the claim that the 1970 proportions are no longer valid. Try with both approaches, the classical and the p -value.

ANSWERS

- H_0 : The number of absences are equally distributed over all days of the week.
 H_A : The number of absences are not equally distributed over all days of the week.
critical value: $\chi^2_{(4,0.05)} = 9.49$ Reject H_0
test statistic: $\chi^2 = 10.40$
 $0.025 < p\text{-value} < 0.05$
- H_0 : The die is fair.
 H_A : The die is not fair.
critical value: $\chi^2_{(5,0.05)} = 11.07$ Fail to Reject H_0
test statistic: $\chi^2 = 4.16$
 $0.50 < p\text{-value} < 0.75$
- H_0 : The distribution in the quality of units is the same for both machines.
 H_A : The distribution in the quality of units is the same for both machines.
critical value: $\chi^2_{(3,0.10)} = 6.25$ Reject H_0
test statistic: $\chi^2 = 6.72$
 $0.010 < p\text{-value} < 0.025$
- H_0 : Sums produced by the pair of dice follow the expected distribution.
 H_A : Sums produced by the pair of dice do not follow the expected distribution.
critical value: $\chi^2_{(10,0.01)} = 23.21$ Fail to reject H_0
test statistic: $\chi^2 = 11.73$
 $0.25 < p\text{-value} < 0.50$
- H_0 : The instructor's claim is valid
 H_A : The instructor's claim is not valid
critical value: $\chi^2_{(2,0.05)} = 5.99$ Fail to reject H_0
test statistic: $\chi^2 = 3.91$
 $0.10 < p\text{-value} < 0.25$
- H_0 : The 1970 proportions are still valid
 H_A : The 1970 proportions are no longer valid
critical value: $\chi^2_{(3,0.01)} = 11.34$ Reject H_0
test statistic: $\chi^2 = 41.51$
 $p\text{-value} < 0.005$