

## MATHEMATICS 201-510-LW

Business Statistics

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### XXIII – Chi-Square Test of Independence

1. A random sample of 1000 was asked whether they voted in the last general election. Here are the results, broken down by age group.

	18-24 years	25-39 years	40-59 years	60 years or older
Voted	46	159	178	159
Did not vote	90	179	118	71

At the 1% level of significance, test the claim that voting status and age are independent. Try with both approaches, the classical and the  $p$ -value.

2. A company operates four machines on two separate shifts daily. The following table gives the number of machine breakdowns recorded in the past 6 months.

	Shift 1	Shift 2
Machine 1	47	96
Machine 2	56	117
Machine 3	27	81
Machine 4	17	94

At the 5% level of significance, test whether the machine causing the breakdown is independent of the shift where the breakdown occurred. Try with both approaches, the classical and the  $p$ -value.

3. A wedding planner took a random sample of 300 men and women were asked “How long should couples date before getting married?” Here are the results.

	Less Than 1 Year	1 Year	1-2 Years	2-3 Years	Longer Than 3 Years
Men	31	45	48	16	10
Women	29	43	51	16	11

At the 5% level of significance, test the claim that a person's response to this question is independent of the person's gender. Try with both approaches, the classical and the  $p$ -value.

4. An employer wants to determine if there is a relationship between an employee's performance in the company's training program and his or her success on the job (as determined by the employer). A random sample of employees was taken, and the following results obtained.

		Performance in Training Program		
		Below average	Average	Above Average
Success in job	Excellent	6	36	48
	Very Good	25	44	37
	Good	30	61	44
	Poor	62	24	8

Using a 2.5% significance level, can we conclude that the performance in training program and success in job are dependent? Try with both approaches, the classical and the p-value.

5. As part of a marketing research, a department store compared the income distribution of its shoppers in three locations. Random sample of shoppers were taken at the three locations, and their income noted. Here are the results.

	Income			
	Under \$15 000	\$15 000 to \$29 999	\$30 000 to \$44 999	\$45 000 or more
Location 1	91	82	74	40
Location 2	109	91	79	87
Location 3	133	111	96	68

Using a 5% level of significance, test the claim that the income of shoppers is independent of location. Try with both approaches, the classical and the p-value.

6. The manager of an assembly process wants to determine whether the number of defective articles manufactured depends on the day of the week the articles are produced. She collected the following information.

Day of the Week	Monday	Tuesday	Wednesday	Thursday	Friday
Nondefective	85	90	95	95	90
Defective	15	10	5	5	10

Using a 10% level of significance, test the claim that the number of defective items is independent of the day of the week. Try with both approaches, the classical and the p-value.

## ANSWERS

- $H_0$ : Voting status is independent of age group.  
 $H_A$ : Voting status is dependent of age group.  
critical value:  $\chi^2_{(3,0.01)} = 11.34$       Reject  $H_0$   
test statistic:  $\chi^2 = 54.58$   
 $p$  – value  $< 0.005$
- $H_0$ : The machine causing the breakdown is independent of the shift where the breakdown occurred.  
 $H_A$ : The machine causing the breakdown is not independent of the shift where the breakdown occurred.  
critical value:  $\chi^2_{(3,0.05)} = 7.81$       Reject  $H_0$   
test statistic:  $\chi^2 = 12.73$   
 $0.005 < p$  – value  $< 0.01$
- $H_0$ : Responses are independent of gender.  
 $H_A$ : Responses are dependent of gender.  
critical value:  $\chi^2_{(4,0.05)} = 9.49$       Fail to reject  $H_0$   
test statistic:  $\chi^2 = 0.251$   
 $0.990 < p$  – value  $< 0.995$
- $H_0$ : Performance in job training is independent of success on job.  
 $H_A$ : Performance in job training is dependent of success on job.  
critical value:  $\chi^2_{(6,0.025)} = 14.45$       Reject  $H_0$   
test statistic:  $\chi^2 = 98.08$   
 $p$  – value  $< 0.005$
- $H_0$ : Income is independent of location.  
 $H_A$ : Income is independent of location.  
critical value:  $\chi^2_{(6,0.05)} = 12.59$       Fail to reject  $H_0$   
test statistic:  $\chi^2 = 12.10$   
 $0.05 < p$  – value  $< 0.1$
- $H_0$  : The number of defective items is independent of the day of the week  
 $H_A$  : They are dependent  
critical value:  $\chi^2_{(4,0.10)} = 7.78$   
test statistic:  $\chi^2 = 8.55$   
 $0.05 < p$  – value  $< 0.10$       Reject  $H_0$