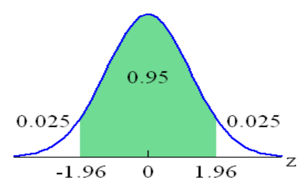
MATHEMATICS 201-510-LW
Business Statistics
Martin Huard
Fall 2008

XXI – Inferences for Independent Samples
SOLUTIONS

1. A supermarket chain is considering opening a new store in either Mathtown or in Mathville. As part of the marketing research, the chain took random sample of households in the two locations, to see if there is a difference in the weekly amount spent on groceries. In Mathtown, the mean weekly grocery bill, for a ramdom sample of 52 households, was \$156.37 with a standard deviation of \$23.12, and in Mathville the mean weekly grocery bill, for a ramdom sample of 45 households, was \$143.24 with a standard deviation of \$15.21.
   a) Construct a 95% confidence interval for the difference in the mean weekly grocery bill of households in Mathtown and in Mathville.

Step 1 Assumptions: $n_{MT} = 52 \geq 30$, $n_{MV} = 45 \geq 30$
The samples are independent.

Step 2 a) Test statistic: z
b) Level of confidence: $1-\alpha = 0.95$ or $\alpha = 0.05$



Step 3 Point estimate: $\overline{x}_{MT} - \overline{x}_{MV} = 156.37 - 143.24 = \$13.13$

Step 4 a) $z_{\frac{\alpha}{2}} = z_{0.025} = 1.96$

b) $E = z_{\frac{\alpha}{2}}\sqrt{\frac{s_{MT}^2}{n_{MT}} + \frac{s_{MV}^2}{n_{MV}}} = 1.96\sqrt{\frac{23.12^2}{52} + \frac{15.21^2}{45}} = 7.70$

c) $\left(\overline{x}_{MT} - \overline{x}_{MV}\right) - E < \mu_{MT} - \mu_{MV} < \left(\overline{x}_{MT} - \overline{x}_{MV}\right) + E$

$13.13 - 7.70 < \mu_{MT} - \mu_{MV} < 13.13 + 7.70$

$5.43 < \mu_{MT} - \mu_{MV} < 20.83$

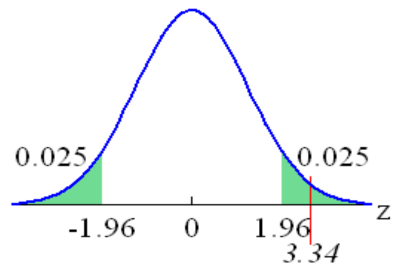
Step 5 The 95% confidence interval for difference in the mean weekly grocery bill of households in Mathtown and in Mathville is \$5.43 to \$20.83.

b) Test at the 5% significance level if there is a difference in the mean weekly grocery bill between the two locations. Try with both approaches, the classical and the *p*-value.

Classical Approach

Step 1 Assumptions: $n_{MT} = 52 \geq 30$, $n_{MV} = 45 \geq 30$

The samples are independent.



Step 2 $H_O: \mu_{MT} - \mu_{MV} = 0$

$H_A: \mu_{MT} - \mu_{MV} \neq 0$

Step 3 a) Test statistic: z

b) Two-tailed test with $\alpha = 0.05$

c) $z_{\frac{\alpha}{2}} = z_{0.025} = 1.96$

Step 4 $$z = \frac{(\bar{x}_{MT} - \bar{x}_{MV}) - (\mu_{MT} - \mu_{MV})}{\sqrt{\frac{s_{MT}^2}{n_{MT}} + \frac{s_{MV}^2}{n_{MV}}}} = \frac{156.37 - 143.24}{\sqrt{\frac{23.12^2}{52} + \frac{15.21^2}{45}}} = 3.34$$

Step 5 a) z is in the critical region

b) Reject H_o.

$\therefore$ There is sufficient evidence at the 5% level of significance to conclude that there is a difference in the mean weekly grocery bill between the two locations.

p-value Approach

Step 1 Assumptions: $n_{MT} = 52 \geq 30$, $n_{MV} = 45 \geq 30$

The samples are independent.

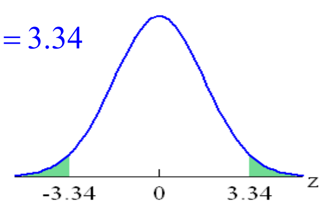
Step 2 $H_O: \mu_{MT} - \mu_{MV} = 0$

$H_A: \mu_{MT} - \mu_{MV} \neq 0$

Step 3 a) Test statistic: z

b) Two-tailed test with $\alpha = 0.05$

Step 4 a) $$z = \frac{(\bar{x}_{MT} - \bar{x}_{MV}) - (\mu_{MT} - \mu_{MV})}{\sqrt{\frac{s_{MT}^2}{n_{MT}} + \frac{s_{MV}^2}{n_{MV}}}} = \frac{156.37 - 143.24}{\sqrt{\frac{23.12^2}{52} + \frac{15.21^2}{45}}} = 3.34$$



b) p-value $= 2P(z < -3.34) = 2(0.0004) = 0.0008$

Step 5 a) p-value $= 0.0008 < \alpha = 0.05$

b) Reject H_o.

$\therefore$ There is sufficient evidence at the 5% level of significance to conclude that there is a difference in the mean weekly grocery bill between the two locations.

2. An experiment was conducted to compare the mean absorption of two drugs (A and B) in specimens of muscle tissue. Seventy-two tissue specimens were randomly divided into two equal groups. Each group was tested with one of the two drugs. The drug A had a mean absorption rate of 7.9 with a standard deviation of 1.1, and drug B had a mean absorption rate of 8.5 with a standard deviation of 1.2.

a) Construct a 90% confidence interval for the difference in the mean absorption rates.

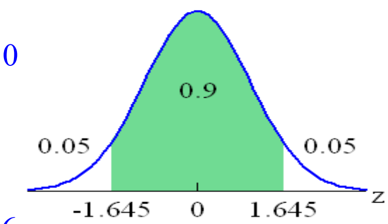
Step 1 Assumptions: $n_A = 36 \geq 30$, $n_B = 36 \geq 30$

The samples are independent.

Step 2 a) Test statistic: z

b) Level of confidence: $1-\alpha = 0.90$ or $\alpha = 0.10$

Step 3 Point estimate: $\bar{x}_B - \bar{x}_A = 8.5 - 7.9 = 0.6$



Step 4 d) $z_{\frac{\alpha}{2}} = z_{0.05} = 1.645$

e) $E = z_{\frac{\alpha}{2}}\sqrt{\frac{s_B^2}{n_B}+\frac{s_A^2}{n_A}} = 1.645\sqrt{\frac{1.1^2}{36}+\frac{1.2^2}{36}} = 0.446$

f) $(\bar{x}_B - \bar{x}_A) - E < \mu_B - \mu_A < (\bar{x}_B - \bar{x}_A) + E$

$0.6 - 0.446 < \mu_B - \mu_A < 0.6 + 0.446$

$0.154 < \mu_B - \mu_A < 1.046$

Step 5 The 90% confidence interval for the difference in the mean absorption rate is between the two drugs is 0.154 to 1.046.

b) Using a 10% level of significance, can you conclude that the absorption rate is different for the two drugs? Try with both approaches, the classical and the *p*-value.

Classical Approach

Step 1 Assumptions: $n_A = 36 \geq 30$, $n_B = 36 \geq 30$

The samples are independent

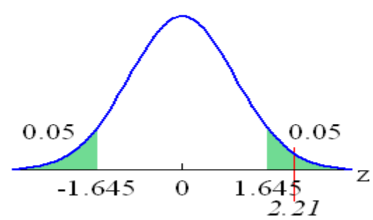
Step 2 $H_O:\ \mu_B - \mu_A = 0$

$H_A:\ \mu_B - \mu_A \neq 0$

Step 3 a) Test statistic: *z*

b) Two-tailed test with α = 0.10

c) $z_{\frac{\alpha}{2}} = z_{0.05} = 1.645$



Step 4 $z = \frac{(\bar{x}_B - \bar{x}_A) - (\mu_B - \mu_A)}{\sqrt{\frac{s_B^2}{n_B}+\frac{s_A^2}{n_A}}} = \frac{8.5-7.9}{\sqrt{\frac{1.1^2}{36}+\frac{1.2^2}{36}}} = 2.21$

Step 5 a) *z* is in the critical region

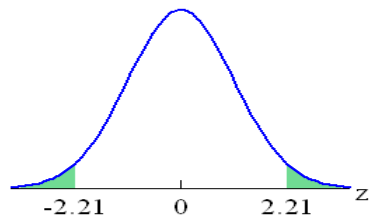
b) Reject H_o.

∴ There is sufficient evidence at the 10% level of significance to conclude that the mean absorption rate is different for the two drugs.

p-value Approach

Step 1 Assumptions: $n_A = 36 \geq 30$, $n_B = 36 \geq 30$

The samples are independent



Step 2 $H_O: \mu_B - \mu_A = 0$

$H_A: \mu_B - \mu_A \neq 0$

Step 3 a) Test statistic: z

b) Two-tailed test with $\alpha = 0.10$

Step 4 a) $z = \dfrac{(\bar{x}_B - \bar{x}_A) - (\mu_B - \mu_A)}{\sqrt{\dfrac{s_B^2}{n_B} + \dfrac{s_A^2}{n_A}}} = \dfrac{8.5 - 7.9}{\sqrt{\dfrac{1.1^2}{36} + \dfrac{1.2^2}{36}}} = 2.21$

b) p-value $= 2P(z > 2.21) = 2(1 - 0.9864) = 0.0272$

Step 5 a) p-value $= 0.0272 < \alpha = 0.10$

b) Reject H_o.

$\therefore$ There is sufficient evidence at the 10% level of significance to conclude that the mean absorption rate is different for the two drugs.

3. "Are intellectuals more likely to wear a beard?" wondered an advertising executive. To answer her question, she asked samples of bearded and clean-shaven men about their level of educational attainment. The following data on years of schooling was obtained.

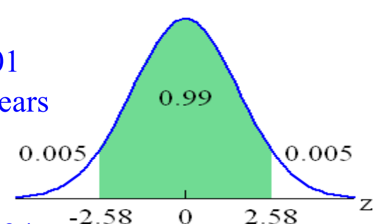
Beard	18	23	12	14	18	16	15	19	12	21	19	8
	21	16	19	20	18	15	14	16	11	12	13	13
	14	19	18	23	19	17	17	12	11			
No Beard	16	11	12	15	14	12	11	19	11	22	13	9
	21	18	19	12	11	14	16	14	13	12	14	14
	12	11	11	15	17	18	21	22	23	14	13	

a) Construct a 99% confidence interval for the difference between the mean schooling of bearded and clean-shaven men.

Step 1 Assumptions: $n_b = 33 \geq 30$, $n_{nb} = 35 \geq 30$, the samples are independent.

Step 2 a) Test statistic: z

b) Level of confidence: $1 - \alpha = 0.99$ or $\alpha = 0.01$



Step 3 Point estimate: $\bar{x}_b - \bar{x}_{nb} = 16.15 - 14.86 = 1.29$ years

Step 4 g) $z_{\frac{\alpha}{2}} = z_{0.005} = 2.58$

h) $E = z_{\frac{\alpha}{2}} \sqrt{\dfrac{s_b^2}{n_b} + \dfrac{s_{nb}^2}{n_{nb}}} = 2.58\sqrt{\dfrac{3.69^2}{33} + \dfrac{3.77^2}{35}} = 2.34$

i) $(\bar{x}_b - \bar{x}_{nb}) - E < \mu_b - \mu_{nb} < (\bar{x}_b - \bar{x}_{nb}) + E$

$1.29 - 2.34 < \mu_n - \mu_{nb} < 1.29 + 2.34$

$-1.05 < \mu_b - \mu_{nb} < 3.63$

Step 5 The 99% confidence interval for the difference in the mean schooling of bearded and clean-shaven men is -1.05 to 3.63 years.

b) Using a 1% level of significance, can you conclude bearded men have more schooling than clean-shaven men? Try with both approaches, the classical and the *p*-value.

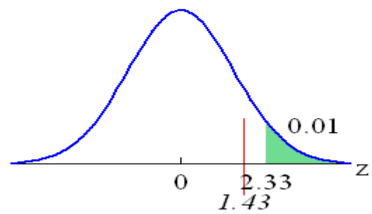
Classical Approach

Step 1 Assumptions: $n_b = 33 \geq 30$, $n_{nb} = 35 \geq 30$

The samples are independent

Step 2 $H_0 : \mu_b - \mu_{nb} = 0$

$H_A : \mu_b - \mu_{nb} > 0$



Step 3 a) Test statistic: z

b) Right-tailed test with $\alpha = 0.01$

c) $z_\alpha = z_{0.01} = 2.33$

Step 4 $$z = \frac{(\bar{x}_b - \bar{x}_{nb}) - (\mu_b - \mu_{nb})}{\sqrt{\frac{s_b^2}{n_b} + \frac{s_{nb}^2}{n_{nb}}}} = \frac{16.15 - 14.86}{\sqrt{\frac{3.69^2}{33} + \frac{3.77^2}{35}}} = 1.43$$

Step 5 a) z is not in the critical region

b) Fail to reject H_0.

$\therefore$ There is not sufficient evidence at the 1% level of significance to conclude that the mean schooling of bearded men is higher than that of clean-shaven men.

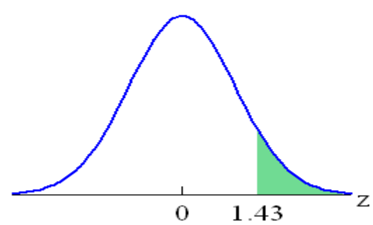
p-value Approach

Step 1 Assumptions: $n_b = 33 \geq 30$, $n_{nb} = 35 \geq 30$

The samples are independent

Step 2 $H_0 : \mu_b - \mu_{nb} = 0$

$H_A : \mu_b - \mu_{nb} > 0$



Step 3 a) Test statistic: z

b) Right-tailed test with $\alpha = 0.01$

Step 4 a) $$z = \frac{(\bar{x}_b - \bar{x}_{nb}) - (\mu_b - \mu_{nb})}{\sqrt{\frac{s_b^2}{n_b} + \frac{s_{nb}^2}{n_{nb}}}} = \frac{16.15 - 14.86}{\sqrt{\frac{3.69^2}{33} + \frac{3.77^2}{35}}} = 1.43$$

b) p-value $= P(z > 1.43) = 1 - 0.9236 = 0.0764$

Step 5 a) p-value $= 0.0764 > \alpha = 0.01$

b) Fail to reject H_0.

$\therefore$ There is not sufficient evidence at the 1% level of significance to conclude that the mean schooling of bearded men is higher than that of clean-shaven men.

4. Bell Canada is considering an incentive plan to encourage its customers to pay their bills promptly. The plan is to offer a 1% discount to those who pay their bill within 5 days instead of the usual 25 days. As an experiment, 20 customers were offered the discount on their November bill, and another 20 customers were not offered the discount. The following results were obtained.

Discount	12	5	4	6	7	21	5	3	4	11
No discount	22	18	3	28	24	23	21	15	22	22

Discount	5	18	2	12	4	19	18	1	23	6
No discount	21	30	2	16	21	30	15	5	20	15

a) Construct a 95% confidence interval for the difference in the mean number of days taken to pay the bill by customers who were offered a discount and the ones who weren't.

Step 1 Assumptions: Populations are normally distributed
The samples are independent.
Variances are equal.

Step 2 a) Test statistic: t with $df = n_N + n_S - 2 = 38$

b) Level of confidence: $1-\alpha = 0.95$ or $\alpha = 0.05$

Step 3 Point estimate: $\bar{x}_N - \bar{x}_D = 18.65 - 9.30 = 9.35$ days

Step 4 a) $t_{\left(df,\frac{\alpha}{2}\right)} = t_{(38,0.025)} = 2.030$

b) $s_p = \sqrt{\dfrac{(n_N-1)s_N^2 + (n_D-1)s_D^2}{n_N+n_D-2}} = \sqrt{\dfrac{19\cdot 7.949^2 + 19\cdot 6.944^2}{38}} = 7.464$

$$E = t_{\left(df,\frac{\alpha}{2}\right)} s_p \sqrt{\frac{1}{n_D} + \frac{1}{n_N}} = 2.030\cdot 7.464\sqrt{\frac{1}{20}+\frac{1}{20}} = 4.626$$

a) $(\bar{x}_N - \bar{x}_D) - E < \mu_N - \mu_D < (\bar{x}_N - \bar{x}_D) + E$

$9.35 - 4.63 < \mu_N - \mu_D < 9.35 - 4.63$

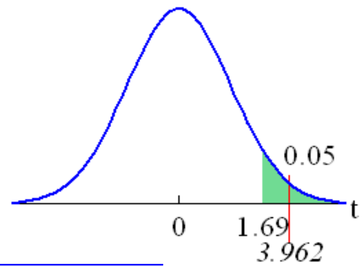
$4.72 < \mu_N - \mu_D < 13.98$

Step 5 The 95% confidence interval for the difference in mean number of days taken to pay the bill by customers who were offered a discount and the ones who weren't is 4.72 to 13.98 days

b) Using a 5% level of significance, can we conclude that customers who were offered the discount paid before those who weren't? Try with both approaches, the classical and the *p*-value.

Classical Approach

Step 1 Assumptions: Populations are normally distributed
The samples are independent.
Variances are equal.



Step 2 $H_0 : \mu_N - \mu_D = 0$

$H_A : \mu_N - \mu_D > 0$

Step 3 a) Test statistic: t with $df = n_N + n_S - 2 = 38$

b) Right-tailed test with $\alpha = 0.05$

c) $t_{(df,\alpha)} = t_{(38,0.05)} = 1.690$

Step 4
$$s_p = \sqrt{\frac{(n_N - 1)s_N^2 + (n_D - 1)s_D^2}{n_N + n_D - 2}} = \sqrt{\frac{19 \cdot 7.949^2 + 19 \cdot 6.944^2}{38}} = 7.464$$

$$t = \frac{(\bar{x}_N - \bar{x}_D) - (\mu_N - \mu_D)}{s_p \sqrt{\frac{1}{n_N} + \frac{1}{n_D}}} = \frac{18.65 - 9.30}{7.464\sqrt{\frac{1}{20} + \frac{1}{20}}} = 3.962$$

Step 5 a) t is in the critical region

b) Reject H_o.

$\therefore$ There is sufficient evidence at the 5% level of significance to conclude that customers who were offered the discount paid before those who weren't.

p-value Approach

Step 1 Assumptions: Populations are normally distributed
The samples are independent.
Variances are equal.

Step 2 $H_0 : \mu_N - \mu_D = 0$

$H_A : \mu_N - \mu_D > 0$

Step 3 a) Test statistic: t with $df = n_N + n_S - 2 = 38$

b) Two-tailed test with $\alpha = 0.05$

Step 4
$$s_p = \sqrt{\frac{(n_N - 1)s_N^2 + (n_D - 1)s_D^2}{n_N + n_D - 2}} = \sqrt{\frac{19 \cdot 7.949^2 + 19 \cdot 6.944^2}{38}} = 7.464$$

$$t = \frac{(\bar{x}_N - \bar{x}_D) - (\mu_N - \mu_D)}{s_p \sqrt{\frac{1}{n_N} + \frac{1}{n_D}}} = \frac{18.65 - 9.30}{7.464\sqrt{\frac{1}{20} + \frac{1}{20}}} = 3.962$$

$p\text{-value} < 0.001$

Step 5 a) $p\text{-value} < 0.001 < \alpha = 0.05$

b) Reject H_o.

$\therefore$ There is sufficient evidence at the 5% level of significance to conclude that customers who were offered the discount paid before those who weren't.

5. The CAA wants to compare the price of gasoline between Quebec and Montreal. For this, two random samples of gas stations were take, one in Montreal and the other in Quebec, and the price for regular gas was noted (in cents per liter), on the same day at the same hour, for each gas station. Here are the results.

Quebec	101.9	100.9	100.9	102.9	99.9
Montreal	99.9	98.9	98.9	101.9	99.9

a) Construct a 99% confidence interval for the difference in the mean price of gas between Quebec and Montreal. Assume that gasoline prices are normally distributed.

Step 1 Assumptions: Populations are normally distributed
The samples are independent.
Variances are equal.

Step 2 a) Test statistic: t with $df = n_Q + n_M - 2 = 8$

b) Level of confidence: $1-\alpha = 0.99$ or $\alpha = 0.01$

Step 3 Point estimate: $\bar{x}_Q - \bar{x}_M = 101.3 - 99.9 = 1.4$ cents/liter

Step 4 a) $t_{\left(df,\frac{\alpha}{2}\right)} = t_{(8,0.005)} = 3.355$

b) $s_p = \sqrt{\dfrac{\left(n_Q-1\right)s_Q^2 + \left(n_M-1\right)s_M^2}{n_Q+n_M-2}} = \sqrt{\dfrac{4\cdot 1.140^2 + 4\cdot 1.225^2}{8}} = 1.183$

$$E = t_{\left(df,\frac{\alpha}{2}\right)} s_p \sqrt{\frac{1}{n_Q} + \frac{1}{n_M}} = 3.355\cdot 1.183\sqrt{\frac{1}{5}+\frac{1}{5}} = 2.511$$

c) $\left(\bar{x}_Q - \bar{x}_M\right) - E < \mu_Q - \mu_M < \left(\bar{x}_Q - \bar{x}_M\right) + E$

$$1.4 - 2.51 < \mu_Q - \mu_M < 1.4 + 2.51$$

$$-1.11 < \mu_T - \mu_I < 3.91$$

Step 5 The 99% confidence interval for the in the mean price of gas between Quebec and Montreal is -1.11 to 3.91 cents per liter.

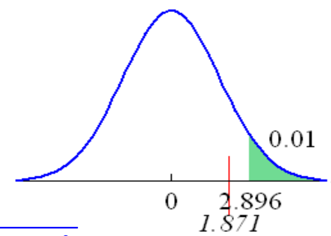
b) Using a 1% level of significance, can we conclude that the mean price of gas is higher in Quebec than in Montreal? Try with both approaches, the classical and the *p*-value. Assume that gasoline prices are normally distributed.

Classical Approach

Step 1 Assumptions: Populations are normally distributed
The samples are independent and the variances are equal.

Step 2 $H_0 : \mu_Q - \mu_M = 0$

$H_A : \mu_Q - \mu_N > 0$



Step 3 a) Test statistic: *t* with $df = n_T + n_I - 2 = 8$

b) Right-tailed test with α = 0.01

c) $t_{(df,\alpha)} = t_{(8,0.01)} = 2.896$

Step 4
$$s_p = \sqrt{\frac{(n_Q - 1)s_Q^2 + (n_M - 1)s_M^2}{n_Q + n_M - 2}} = \sqrt{\frac{4\cdot 1.140^2 + 4\cdot 1.225^2}{8}} = 1.183$$

$$t = \frac{(\bar{x}_Q - \bar{x}_M) - (\mu_Q - \mu_M)}{s_p\sqrt{\frac{1}{n_Q} + \frac{1}{n_M}}} = \frac{101.3 - 99.9}{1.183\sqrt{\frac{1}{5} + \frac{1}{5}}} = 1.871$$

Step 5 a) *t* is not in the critical region

b) Fail to reject H_o.

∴ There is not sufficient evidence at the 1% level of significance to conclude that the mean price of gas is higher in Quebec than in Montreal.

p-value Approach

Step 1 Assumptions: Populations are normally distributed
The samples are independent and the variances are equal.

Step 2 $H_0 : \mu_Q - \mu_M = 0$

$H_A : \mu_Q - \mu_N > 0$

Step 3 a) Test statistic: *t* with $df = n_T + n_I - 2 = 8$

b) Right-tailed test with α = 0.01

Step 4
$$s_p = \sqrt{\frac{(n_Q - 1)s_Q^2 + (n_M - 1)s_M^2}{n_Q + n_M - 2}} = \sqrt{\frac{4\cdot 1.140^2 + 4\cdot 1.225^2}{8}} = 1.183$$

$$t = \frac{(\bar{x}_Q - \bar{x}_M) - (\mu_Q - \mu_M)}{s_p\sqrt{\frac{1}{n_Q} + \frac{1}{n_M}}} = \frac{101.3 - 99.9}{1.183\sqrt{\frac{1}{5} + \frac{1}{5}}} = 1.871$$

$0.047 < p\text{-value} < 0.055$

Step 5 a) p-value > 0.047 > α = 0.01

b) Fail to reject H_o.

∴ There is not sufficient evidence at the 1% level of significance to conclude that the mean price of gas is higher in Quebec than in Montreal.

6. The viscosity of two different brands of car oil are measured and the following data resulted:

Brand 1	10.02	10.33	10.89	10.66	10.63	10.85	10.70	10.44	10.76	10.61
Brand 2	10.19	10.43	10.57	10.30	10.65	10.36	10.36			

Assume that the viscosity of car oils are normally distributed.

a) Construct a 90% confidence interval for the difference in the mean viscosity of the two brands.

Step 1 Assumptions: Populations are normally distributed
The samples are independent.
Variances are equal.

Step 2 a) Test statistic: t with $df = n_B + n_W - 2 = 15$

b) Level of confidence: $1-\alpha = 0.90$ or $\alpha = 0.10$

Step 3 Point estimate: $\bar{x}_1 - \bar{x}_2 = 10.589 - 10.409 = 0.180$

Step 4 a) $t_{\left(df,\frac{\alpha}{2}\right)} = t_{(15,0.05)} = 1.753$

b) $s_p = \sqrt{\dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\dfrac{9\cdot 0.2626^2 + 6\cdot 0.1576^2}{15}} = 0.2265$

$E = t_{\left(df,\frac{\alpha}{2}\right)} s_p \sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}} = 1.753 \cdot 0.2265 \sqrt{\dfrac{1}{10} + \dfrac{1}{7}} = 0.1957$

c) $(\bar{x}_1 - \bar{x}_2) - E < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + E$

$0.180 - 0.196 < \mu_1 - \mu_2 < 0.180 + 0.196$

$-0.014 < \mu_1 - \mu_2 < 0.376$

Step 5 The 90% confidence interval for the difference in the mean viscosity of the two brands is -0.014 to 0.376.

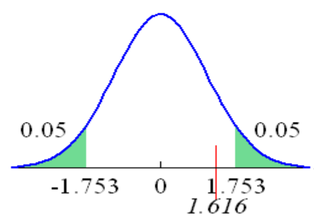
b) Using a 10% level of significance, can we conclude that the two brands do not have the same viscosity? Try with both approaches, the classical and the *p*-value.

Classical Approach

Step 1 Assumptions: Populations are normally distributed
The samples are independent and the variances are equal.

Step 2 $H_0: \mu_1 - \mu_2 = 0$

$H_A: \mu_1 - \mu_2 \neq 0$



Step 3 a) Test statistic: *t* with $df = n_B + n_W - 2 = 15$

b) Two-tailed test with $\alpha = 0.10$

c) $t_{\left(df, \frac{\alpha}{2}\right)} = t_{(16,0.05)} = 1.753$

Step 4
$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{9 \cdot 0.2626^2 + 6 \cdot 0.1576^2}{15}} = 0.2265$$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{10.589 - 10.409}{0.2265\sqrt{\frac{1}{10} + \frac{1}{7}}} = 1.616$$

Step 5 a) *t* is not in the critical region

b) Fail to reject H_0.

∴ There is not sufficient evidence at the 10% level of significance to conclude that the two brands do not have the same viscosity.

p-value Approach

Step 1 Assumptions: Populations are normally distributed
The samples are independent and the variances are equal.

Step 2 $H_0: \mu_1 - \mu_2 = 0$

$H_A: \mu_1 - \mu_2 \neq 0$

Step 3 a) Test statistic: *t* with $df = n_B + n_W - 2 = 15$

b) Two-tailed test with $\alpha = 0.10$

Step 4
$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{9 \cdot 0.2626^2 + 6 \cdot 0.1576^2}{15}} = 0.2265$$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{10.589 - 10.409}{0.2265\sqrt{\frac{1}{10} + \frac{1}{7}}} = 1.616$$

$$2 \cdot 0.055 < p\text{-value} < 2 \cdot 0.065$$

$$0.110 < p\text{-value} < 0.130$$

Step 5 a) p-value $> 0.110 > \alpha = 0.10$

b) Fail to reject H_0.

∴ There is not sufficient evidence at the 10% level of significance to conclude that the two brands do not have the same viscosity.