MATHEMATICS 201-510-LW
Business Statistics
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XX – Inferences for Dependent Samples
SOLUTIONS

1. A company sent seven of its employees to attend a course on building self-confidence. These employees were evaluated for their self-confidence before and after attending this course. The following table gives the scores (on a scale of 1 to 15, 1 being the lowest and 15 being the highest score) of these employees before and after they attended the course. Assume that the population of paired differences has a normal distribution

Before	8	5	4	9	6	8	5
After	10	7	5	11	6	7	9
A-B	2	2	1	2	0	-1	4

a) Construct a 95% confidence interval for the mean of the population paired difference.

Step 1 Assumptions: The sampled populations are normally distributed.

Step 2 a) Test statistic: t with $df = n-1 = 6$

b) Level of confidence: $1-\alpha = 0.95$ or $\alpha = 0.05$

Step 3 Point estimate: $\bar{d} = 1.429$

Step 4 a) $t_{\left(df, \frac{\alpha}{2}\right)} = t_{(6, 0.025)} = 2.447$

b) $E = t_{\left(df, \frac{\alpha}{2}\right)} \frac{s_d}{\sqrt{n}} = 2.447 \frac{1.618}{\sqrt{7}} = 1.50$

c) $\bar{d} - E < \mu_d < \bar{d} + E$

$1.43 - 1.50 < \mu_d < 1.43 + 1.50$

$-0.07 < \mu_d < 2.93$

Step 5 The 95% confidence interval for the mean difference in the self-confidence score is -0.07 to 2.93.

b) Test at the 1% significance level if attending this course increases then mean score of employees. Try with both approaches, the classical and the *p*-value.

Classical Approach

Step 1 Assumptions: The populations are normally distributed.

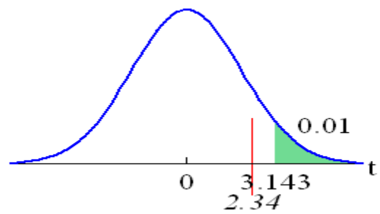
Step 2 $H_O:\ \mu_d = 0$

$H_A:\ \mu_d > 0$

Step 3 a) Test statistic: t with $df = n-1 = 6$

b) Right-tailed test with $\alpha = 0.01$

c) $t_{(df,\alpha)} = t_{(6,0.01)} = 3.143$



Step 4 $t = \dfrac{\bar{d}-\mu_d}{\frac{s_d}{\sqrt{n}}} = \dfrac{1.429-0}{\frac{1.618}{\sqrt{7}}} = 2.34$

Step 5 a) t is not in the critical region

b) Fail to reject H_o.

$\therefore$ There is not sufficient evidence at the 1% level of significance to conclude that attendance to this course increases the mean score of employees.

p-value approach

Step 1 Assumptions: The populations are normally distributed

Step 2 $H_O:\ \mu_d = 0$

$H_A:\ \mu_d > 0$

Step 3 a) Test statistic: t with $df = n-1 = 6$

b) Right-tailed test with $\alpha = 0.01$

Step 4 a) $t = \dfrac{\bar{d}-\mu_d}{\frac{s_d}{\sqrt{n}}} = \dfrac{1.429-0}{\frac{1.618}{\sqrt{7}}} = 2.34$

b) $0.027 < p\text{-value} < 0.031$

Step 5 a) $p\text{-value} > 0.027 > \alpha = 0.01$

b) Fail to reject H_o.

$\therefore$ There is not sufficient evidence at the 1% level of significance to conclude that attendance to this course increases the mean score of employees.

2. A private agency claims that the crash course it offers significantly increases the writing speed of secretaries. The following table gives the scores of eight secretaries before and after they attended the course.

Before	81	75	89	91	65	70	90	69
After	97	72	93	110	78	69	115	75
A-B	16	-3	4	19	13	-1	25	6

Assume that the writing speeds before and after attending the course are normally distributed.

a) Construct a 90% confidence interval for the mean increase in writing speed of secretaries.

Step 1 Assumptions: The sampled populations are normally distributed.

Step 2 a) Test statistic: t with $df = n-1 = 7$

b) Level of confidence: $1-\alpha = 0.90$ or $\alpha = 0.10$

Step 3 Point estimate: $\bar{d} = 9.875$

Step 4 a) $t_{\left(df, \frac{\alpha}{2}\right)} = t_{(7, 0.05)} = 1.895$

b) $E = t_{\left(df, \frac{\alpha}{2}\right)} \frac{s_d}{\sqrt{n}} = 1.895 \frac{9.949}{\sqrt{7}} = 6.664$

c) $\bar{d} - E < \mu_d < \bar{d} + E$

$9.87 - 6.66 < \mu_d < 9.87 + 6.66$

$3.21 < \mu_d < 16.55$

Step 5 The 90% confidence interval for the mean increase in writing speed of secretaries is 3.21 to 16.55.

b) Using a 5% level of significance, can you conclude if attending this course increases the writing speed of secretaries? Try with both approaches, the classical and the p-value.

Classical Approach

Step 1 Assumptions: The populations are normally distributed.

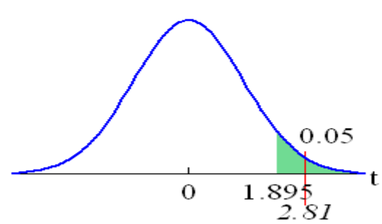
Step 2 $H_O:\ \mu_d = 0$

$H_A:\ \mu_d > 0$

Step 3 a) Test statistic: t with $df = n-1 = 7$

b) Right-tailed test with $\alpha = 0.05$

c) $t_{(df, \alpha)} = t_{(7, 0.05)} = 1.895$



Step 4 $t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} = \frac{9.875 - 0}{\frac{9.949}{\sqrt{8}}} = 2.81$

Step 5 a) t is in the critical region

b) Reject H_o.

∴ There is sufficient evidence at the 5% level of significance to conclude that attendance to this course increases the mean writing speed of secretaries.

p-value approach

Step 1 Assumptions: The populations are normally distributed

Step 2 $H_O:\ \mu_d = 0$

$H_A:\ \mu_d > 0$

Step 3 a) Test statistic: t with $df = n-1 = 7$

b) Right-tailed test with $\alpha = 0.05$

Step 4 a) $t = \dfrac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} = \dfrac{9.875 - 0}{\frac{9.949}{\sqrt{8}}} = 2.81$

b) $0.011 < p\text{-value} < 0.013$

Step 5 a) $p\text{-value} < 0.013 < \alpha = 0.05$

b) Reject H_o.

∴ There is sufficient evidence at the 5% level of significance to conclude that attendance to this course increases the mean writing speed of secretaries.

3. An industrial safety program was recently instituted at all assembly plant in the aeronautical industry. The average weekly losses (averaged over one month) in man-hours due to accidents in 5 similar plants both before and after the program are given below. Assume that weekly losses in man-hours due to accidents are normally distributed.

Before	30.5	24	28	23.5	16
After	24	25.5	21	18	14.5

a) Construct a 95% confidence interval for the difference in the number of weekly losses in man-hours due to accidents.

Step 1 Assumptions: The sampled populations are normally distributed.

Step 2 a) Test statistic: t with $df = n-1 = 4$

b) Level of confidence: $1-\alpha = 0.95$ or $\alpha = 0.05$

Step 3 Point estimate: $\bar{d} = 3.8$ hours

Step 4 a) $t_{\left(df,\frac{\alpha}{2}\right)} = t_{(4,0.025)} = 2.776$

b) $E = t_{\left(df,\frac{\alpha}{2}\right)} \frac{s_d}{\sqrt{n}} = 2.776 \frac{3.6674}{\sqrt{5}} = 4.554$

c) $\bar{d} - E < \mu_d < \bar{d} + E$

$3.80 - 4.55 < \mu_d < 3.80 + 4.55$

$-0.75 < \mu_d < 8.35$

Step 5 The 95% confidence interval for the mean difference in the number of weekly losses in man-hours due to accidents -0.75 to 8.35 hours.

b) Using a 5% level of significance, can you conclude that the safety program reduced the average weekly losses in man-hours due to accidents? Try with both approaches, the classical and the *p*-value.

Classical Approach

Step 1 Assumptions: The populations are normally distributed.

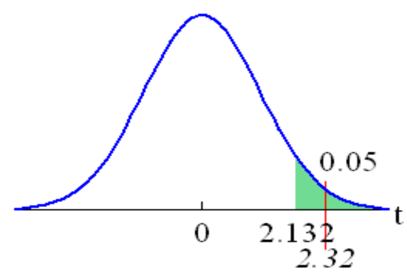
Step 2 $H_O:\ \mu_d = 0$

$H_A:\ \mu_d \neq 0$

Step 3 a) Test statistic: t with $df = n-1 = 4$

b) Right-tailed test with $\alpha = 0.05$

c) $t_{(df,\alpha)} = t_{(4,0.05)} = 2.132$



Step 4 $t = \dfrac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} = \dfrac{3.8 - 0}{\frac{3.6674}{\sqrt{5}}} = 2.32$

Step 5 a) t is in the critical region

b) Reject H_o.

∴ There is sufficient evidence at the 5% level of significance to conclude that the safety program reduced the average weekly losses in man-hours due to accidents.

p-value approach

Step 1 Assumptions: The populations are normally distributed

Step 2 $H_O:\ \mu_d = 0$

$H_A:\ \mu_d \neq 0$

Step 3 a) Test statistic: t with $df = n-1 = 4$

b) Right-tailed test with $\alpha = 0.05$

Step 4 a) $t = \dfrac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} = \dfrac{3.8 - 0}{\frac{3.6674}{\sqrt{5}}} = 2.32$

b) $0.037 < p\text{-value} < 0.041$

Step 5 a) $p\text{-value} < 0.041 < \alpha = 0.05$

b) Reject H_o.

∴ There is sufficient evidence at the 5% level of significance to conclude that the safety program reduced the average weekly losses in man-hours due to accidents.

4. A garage wants to compare the wearing quality of tires, equally priced, produced by two different companies. A random sample of cars was taken, where a tire of brand A and a tire of Brand B were installed on each car. Here are the amounts of wear, in mm, (after 20000 km). Assume that the amounts of wear are normally distributed.

Car	1	2	3	4	5	6	7	8	9
Brand A	2.81	1.53	1.76	2.51	2.51	1.98	2.26	2.36	1.66
Brand B	2.59	1.69	1.7	2.74	2.39	1.95	2.03	2.83	2.06
$d = B - A$	-0.22	0.16	-0.06	0.23	-0.12	-0.03	-0.23	0.47	0.4

Car	10	11	12	13	14	15	16	17
Brand A	1.91	1.31	1.95	2.03	2.3	1.51	3	1.92
Brand B	1.68	1.61	2.29	1.76	2.48	1.72	2.81	2.38
$d = B - A$	-0.23	0.30	0.34	-0.27	0.18	0.21	-0.19	0.46

a) Construct a 99% confidence interval for the mean difference in the amount of wear between Brand A and Brand B.

Step 1 Assumptions: The sampled populations are normally distributed.

Step 2 a) Test statistic: t with $df = n-1 = 16$

b) Level of confidence: $1-\alpha = 0.99$ or $\alpha = 0.01$

Step 3 Point estimate: $\bar{d} = 0.082$ mm

Step 4 a) $t_{\left(df,\frac{\alpha}{2}\right)} = t_{(16,0.005)} = 2.921$

b) $E = t_{\left(df,\frac{\alpha}{2}\right)} \frac{s}{\sqrt{n}} = 2.921 \frac{0.2646}{\sqrt{17}} = 0.187$

c) $\bar{d} - E < \mu_d < \bar{d} + E$

$0.082 - 0.187 < \mu_d < 0.082 + 0.187$

$-0.105 < \mu_d < 0.270$

Step 5 The 99% confidence interval for the mean difference in the amount of wear between Brand A and Brand B is -0.105 mm to 0.270 mm

b) Using a 10% level of significance, can you conclude that the two brands do not have the same amount of wear? Try with both approaches, the classical and the p-value.

Classical Approach

Step 1 Assumptions: The populations are normally distributed.

Step 2 $H_O: \mu_d = 0$

$H_A: \mu_d \neq 0$

Step 3 a) Test statistic: t with $df = n-1 = 16$

b) Two-tailed test with $\alpha = 0.10$

c) $t_{\left(df,\frac{\alpha}{2}\right)} = t_{(16,0.05)} = 1.746$

Step 4 $t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} = \frac{0.082 - 0}{\frac{0.2646}{\sqrt{17}}} = 1.28$

Step 5 a) t is not in the critical region

b) Fail to reject H_o.

∴ There is insufficient evidence at the 10% level of significance to conclude that there is a difference in the amount of wear between Brand A and Brand B.

p-value approach

Step 1 Assumptions: The populations are normally distributed

Step 2 $H_O: \mu_d = 0$

$H_A: \mu_d \neq 0$

Step 3 a) Test statistic: t with $df = n-1 = 16$

b) Two-tailed test with $\alpha = 0.10$

Step 4 a) $t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} = \frac{0.082 - 0}{\frac{0.2646}{\sqrt{17}}} = 1.28$

b) $2 \cdot 0.105 < p\text{-value} < 2 \cdot 0.124$

$0.210 < p\text{-value} < 0.248$

Step 5 a) $p\text{-value} > 0.210 > \alpha = 0.10$

b) Fail to reject H_o.

$\therefore$ There is insufficient evidence at the 10% level of significance to conclude that there is a difference in the amount of wear between Brand A and Brand B.

5. The manufacturer of a gas additive claims that his product increases mileage. A random sample of 8 cars was taken, where the mileage (l/100km) was noted without and with the additive. Assume that mileage is normally distributed.

Without	9.87	12.13	15.10	8.42	9.05	10.21	13.49	11.05
With	9.12	11.14	12.61	8.55	7.12	9.21	12.14	12.56

a) Construct a 90% confidence interval for the mean difference in mileage without and with the additive.

Step 1 Assumptions: The sampled populations are normally distributed.

Step 2 a) Test statistic: t with $df = n-1 = 7$

b) Level of confidence: $1-\alpha = 0.90$ or $\alpha = 0.10$

Step 3 Point estimate: $\bar{d} = 0.859$

Step 4 a) $t_{\left(df, \frac{\alpha}{2}\right)} = t_{(7, 0.05)} = 1.895$

b) $E = t_{\left(df, \frac{\alpha}{2}\right)} \frac{s_d}{\sqrt{n}} = 1.895 \frac{1.2356}{\sqrt{8}} = 0.828$

c) $\bar{d} - E < \mu_d < \bar{d} + E$

$0.859 - 0.828 < \mu_d < 0.859 + 0.828$

$0.031 < \mu_d < 1.687$

Step 5 The 90% confidence interval for the mean difference in mileage without and with the additive is 0.031 to 1.687 liters per 100 km.

b) Test the manufacturer's claim using a 10% level of significance. Try with both approaches, the classical and the *p*-value.

Classical Approach

Step 1 Assumptions: The populations are normally distributed.

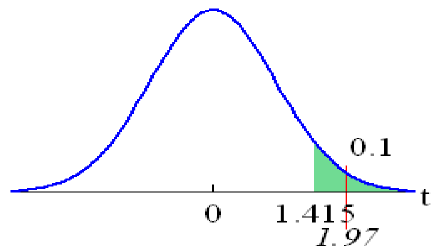
Step 2 $H_O: \mu_d = 0$

$H_A: \mu_d > 0$

Step 3 a) Test statistic: t with $df = n - 1 = 7$

b) Right-tailed test with α = 0.10

c) $t_{(df,\alpha)} = t_{(7,0.1)} = 1.415$



Step 4 $t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} = \frac{0.859 - 0}{\frac{1.2356}{\sqrt{8}}} = 1.97$

Step 5 a) t is in the critical region

b) Reject H_o.

∴ There is sufficient evidence at the 5% level of significance to conclude that the gas additive increases mileage.

p-value approach

Step 1 Assumptions: The populations are normally distributed

Step 2 $H_O: \mu_d = 0$

$H_A: \mu_d > 0$

Step 3 a) Test statistic: t with $df = n - 1 = 7$

b) Two-tailed test with α = 0.10

Step 4 a) $t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} = \frac{0.859 - 0}{\frac{1.2356}{\sqrt{8}}} = 1.97$

b) $0.043 < p\text{-value} < 0.050$

Step 5 a) $p\text{-value} < 0.050 < \alpha = 0.10$

b) Reject H_o.

∴ There is sufficient evidence at the 5% level of significance to conclude that the gas additive increases mileage.