MATHEMATICS 201-510-LW
Business Statistics
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Fall 2008

XII – Hypothesis Testing for μ (large samples)
SOLUTIONS

1. Write the null and alternative hypotheses for each of the following examples. Determine if each is a case of a two-tailed, a left-tailed, or a right-tailed test.
   a) To test whether or not the mean price of houses in Quebec is greater than $143000.
      $H_0 : \mu = \$143000$ Right-tailed
      $H_A : \mu > \$143000$
   b) To test if the mean number of hours spent working per week by college students who hold jobs is different from 15 hours.
      $H_0 : \mu = 15$ hours Two-tailed
      $H_A : \mu \neq 15$ hours
   c) To test whether the mean life of a particular brand of auto batteries is less than 45 months.
      $H_0 : \mu = 45$ months Left-tailed
      $H_A : \mu < 45$ months
   d) To test if the mean amount of time spent doing homework by all fourth-graders is different from 5 hours a week.
      $H_0 : \mu = 5$ hours Two-tailed
      $H_A : \mu \neq 5$ hours
   e) To test if the mean age of all college students is different from 18 years.
      $H_0 : \mu = 18$ years Two-tailed
      $H_A : \mu \neq 18$ years

2. Consider $H_0 : \mu = 20$ versus $H_A : \mu < 20$
   a) What type of error would you make if the null hypothesis is actually false and you fail to reject it? Type II error
   b) What type of error would you make if the null hypothesis is actually true and you reject it? Type I error

3. According to a Pharmacist, Quebecers spent an average of \$220 per person on prescription drugs in 2006. A recent survey of 300 randomly chosen Quebecers showed that they spent an average of \$235 per person on prescription drugs with a standard deviation of \$90. Test at the 2.5% significance level whether the mean amount currently spent on prescription drugs by all Quebecers exceeds \$220 per person. Use the classical approach.

Step 1 Assumptions: $n = 300 \geq 30$

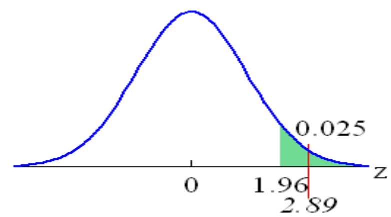
Step 2 $H_0: \mu = \$220$

$H_A: \mu > \$220$

Step 3 a) Test statistic: z

b) Right-tailed test with $\alpha = 0.025$

c) $z_\alpha = z_{0.025} = 0.96$



Step 4 $z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{235 - 220}{\frac{90}{\sqrt{300}}} = 2.89$

Step 5 a) z is in the critical region

b) Reject H_o.

∴ There is sufficient evidence at the 2.5% level of significance to conclude that the mean amount currently spent on prescription drugs by all Quebecers exceed \$220 per person.

4. According to *Statistics Canada*, the average family income in Canada was \$76 100 in 2004. A recently taken sample of 1200 Canadian families yielded a mean income of \$77 152 with a sample standard deviation of \$16 850. Using the 2% significance level, can you conclude that the mean family income in Canada has changed since 2004? Use the classical approach.

Step 1 Assumptions: $n = 1200 \geq 30$

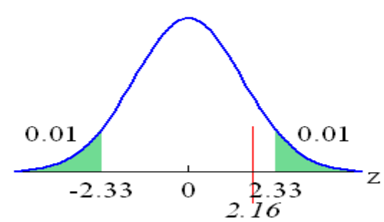
Step 2 $H_0: \mu = \$76100$

$H_A: \mu \neq \$76100$

Step 3 a) Test statistic: z

b) Two-tailed test with $\alpha = 0.02$

c) $z_{\frac{\alpha}{2}} = z_{0.01} = 2.33$



Step 4 $z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{77152 - 76100}{\frac{16850}{\sqrt{1200}}} = 2.16$

Step 5 a) z is not in the critical region

b) Fail to reject H_o.

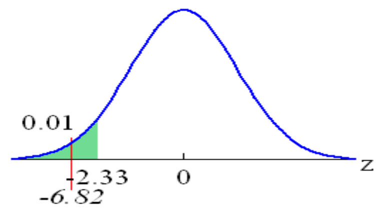
∴ There is not sufficient evidence at the 2% level of significance to conclude that the mean family income in Canada has changed since 2004.

5. A study conducted a few years ago claims that adult males spend an average of 11 hours a week watching sports on television. A recent sample of 100 adult males showed that the mean time they spend per week watching television is 9.50 hours with a standard deviation of 2.2 hours. Test at the 1% significance level if currently all adult males spend less than 11 hours watching sports on television. Use the classical approach.

Step 1 Assumptions: $n = 100 \geq 30$

Step 2 $H_0:\ \mu = 11$ hours

$H_A:\ \mu < 11$ hours



Step 3 a) Test statistic: z

b) Left-tailed test with $\alpha = 0.01$

c) $z_\alpha = z_{0.01} = 2.33$

Step 4 $z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{9.5 - 11}{\frac{2.2}{\sqrt{100}}} = -6.82$

Step 5 a) z is in the critical region

b) Reject H_o.

∴ There is sufficient evidence at the 1% level of significance to conclude that adult males spend less than 11 hours watching sports on television.

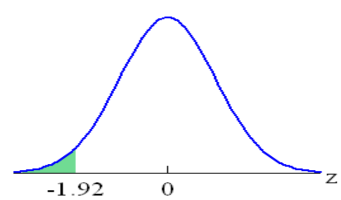
6. A telephone company claims that the mean duration of all long-distance phone calls made by its residential customers is 10 minutes. A random of 100 long-distance calls made by its residential customers taken from the records of this company showed that the mean duration of calls for this sample is 9.0 minutes with a standard deviation of 5.2 minutes. Test the whether the mean duration of all long-distance calls is less than 10 minutes.

(a) at the 2% level of significance

Step 1 Assumptions: $n = 100 \geq 30$

Step 2 $H_0: \mu = 10$ minutes

$H_A: \mu < 10$ minutes



Step 3 a) Test statistic: z

b) Left-tailed test with $\alpha = 0.02$

Step 4 a) $z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{9.0 - 10.0}{\frac{5.2}{\sqrt{100}}} = -1.92$

b) p-value = $P(z < -1.92) = 0.0274$

Step 5 a) p-value = 0.0274 > α = 0.02

b) Fail to reject H_o.

∴ There is not sufficient evidence at the 2% level of significance to conclude that the mean duration of all long-distance calls is less than 10 minutes.

(b) at the 5% level of significance

Step 5 a) p-value = 0.0274 < α = 0.05

b) Reject H_o.

∴ There is sufficient evidence at the 5% level of significance to conclude that the mean duration of long-distance calls is less than 10 minutes.

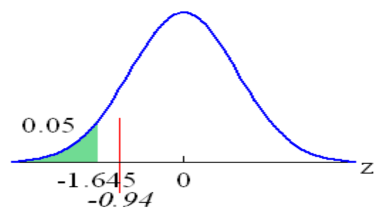
7. For a population of humans to sustain itself, there must be an average of just of over two births for each woman of reproductive age. The fertility rate varies substantially from within one nation to the next. A journalist recently pointed out in *Fortune* magazine that the average for Japan has dropped to 1.5 births for each woman of reproductive age, which could reduce Japan's population from 135 million in 1997 to 50 million by the end of the 21st century. Suppose a random sample of 200 Japanese women of reproductive age is taken in 2002, and the sample mean fertility rate is measured as 1.45 with a standard deviation of 0.75. Did the rate decline? Use a 5% level of significance, with

   a) the classical approach

   Step 1 Assumptions: $n = 200 \geq 30$

   Step 2 $H_0:\ \mu = 1.5$ births

   $H_A:\ \mu < 1.5$ births



   Step 3 a) Test statistic: z

   b) Left-tailed test with $\alpha = 0.05$

   c) $z_\alpha = z_{0.05} = 1.645$

   Step 4 $z = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \dfrac{1.45 - 1.5}{\frac{0.75}{\sqrt{200}}} = -0.94$

   Step 5 a) z is not in the critical region

   b) Fail to reject H_o.

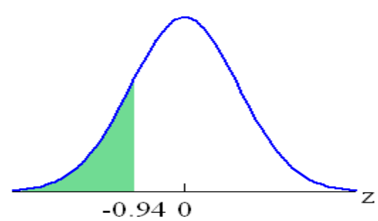
   ∴ There is not sufficient evidence at the 5% level of significance to conclude that the birth rate declined.

   b) the p-value approach

   Step 1 Assumptions: $n = 200 \geq 30$

   Step 2 $H_0:\ \mu = 1.5$ births

   $H_A:\ \mu < 1.5$ births



   Step 3 a) Test statistic: z

   b) Left-tailed test with $\alpha = 0.05$

   Step 4 a) $z = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \dfrac{1.45 - 1.5}{\frac{0.75}{\sqrt{200}}} = -0.94$

   b) p-value $= P(z < -0.94) = 0.1736$

   Step 5 a) p-value $= 0.1736 > \alpha = 0.05$

   b) Fail to reject H_o.

   ∴ There is not sufficient evidence at the 5% level of significance to conclude that the birth rate declined.

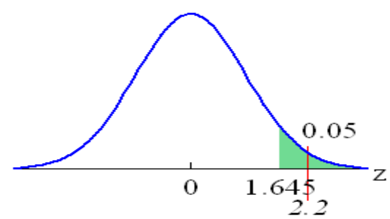
8. A politician claims that the mean number of hours Canadians worked per week in a typical week is greater than 40 hours. A sample of 324 Canadians workers produced a mean of 41.3 hours per week with a standard deviation of 10.63 hours per week. Test the politician's claim at the 5% level of significance using

a) the classical approach

Step 1 Assumptions: $n = 324 \geq 30$

Step 2 $H_0 : \mu = 40$ hours

$H_A : \mu > 40$ hours



Step 3 a) Test statistic: z

b) Right-tailed test with $\alpha = 0.05$

c) $z_\alpha = z_{0.05} = 1.645$

Step 4 $z = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \dfrac{41.3 - 40}{\frac{10.63}{\sqrt{324}}} = 2.20$

Step 5 a) z is in the critical region

b) Reject H_o.

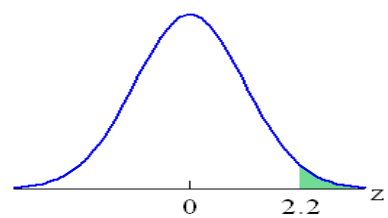
∴ There is sufficient evidence at the 5% level of significance to conclude that Canadians work, on average, more than 40 hours a week.

b) the p-value approach

Step 1 Assumptions: $n = 324 \geq 30$

Step 2 $H_0 : \mu = 40$ hours

$H_A : \mu > 40$ hours



Step 3 a) Test statistic: z

b) Right-tailed test with $\alpha = 0.05$

Step 4 a) $z = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \dfrac{41.3 - 40}{\frac{10.63}{\sqrt{324}}} = 2.20$

b) p-value = $P(z > 2.20) = 1 - 0.9861 = 0.0139$

Step 5 a) p-value = $0.0139 < \alpha = 0.05$

b) Reject H_o.

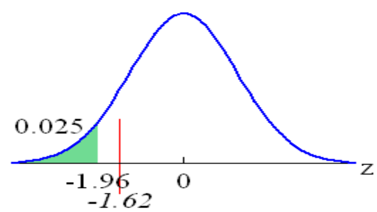
∴ There is sufficient evidence at the 5% level of significance to conclude that Canadians work, on average, more than 40 hours a week.

9. The customers at a bank complained about long lines and the time they had to spend waiting for service. It is known that the customers at this bank had to wait 8 minutes, on average, before being served. The management made some changes to reduce the waiting time for its customers. A sample of 32 customers taken after these changes were made produced a mean waiting time of 7.4 minutes with a standard deviation of 2.1 minutes. Using this sample mean, the bank manager displayed a huge banner inside the bank mentioning that the mean waiting time for customer has been reduced by new changes. Do you think the manager's claim is justifiable? Use a 2.5% level of significance along with the classical approach.

Step 1 Assumptions: $n = 32 \geq 30$

Step 2 $H_0 : \mu = 8$ minutes

$H_A : \mu < 8$ minutes



Step 3 a) Test statistic: z

b) Right-tailed test with α = 0.025

c) $z_\alpha = z_{0.025} = 1.96$

Step 4 $z = \frac{\overline{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{41.3 - 40}{\frac{10.63}{\sqrt{324}}} = -1.62$

Step 5 a) z is not in the critical region

b) Fail to reject H_0.

∴ There is not sufficient evidence at the 5% level of significance to conclude that the waiting times have been reduced. Thus, the manager's claim is not justifiable.

10. A sample of 225 parents were asked how much time they spent per week on school work or school-related activities. This sample produced a mean of 5.6 hours per week, with a standard deviation of 4.4 hours. At the 0.01 level of significance, test the claim that the mean number of hours spent by parents on school work or school-related activities is 5 hours per week, using

a) the classical approach

Step 1 Assumptions: $n = 225 \geq 30$

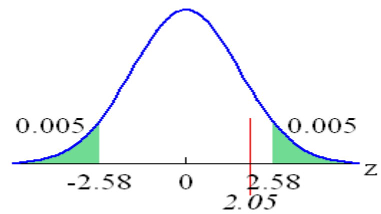
Step 2 $H_0 : \mu = 5$ hours

$H_A : \mu \neq 5$ hours

Step 3 a) Test statistic: z

b) Two-tailed test with $\alpha = 0.01$

c) $z_{\frac{\alpha}{2}} = z_{0.005} = 2.58$



Step 4 $z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{5.6 - 5}{\frac{4.4}{\sqrt{225}}} = 2.05$

Step 5 a) z is not in the critical region

b) Fail to reject H_o.

∴ There is not sufficient evidence at the 5% level of significance to conclude that the mean number of hours spent by parents on school work or school-related activities is different than 5 hours per week.

b) the p-value approach

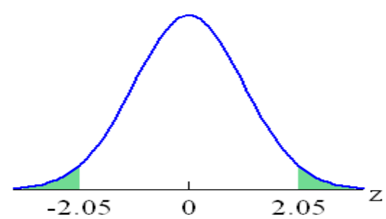
Step 1 Assumptions: $n = 225 \geq 30$

Step 2 $H_0 : \mu = 5$ hours

$H_A : \mu \neq 5$ hours

Step 3 a) Test statistic: z

b) Two-tailed test with $\alpha = 0.01$



Step 4 a) $z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{5.6 - 5}{\frac{4.4}{\sqrt{225}}} = 2.05$

b) p-value $= 2P(z < -2.05) = 2 \cdot 0.0202 = 0.0404$

Step 5 a) p-value $= 0.0404 > \alpha = 0.01$

b) Fail to reject H_o.

∴ There is not sufficient evidence at the 5% level of significance to conclude that the mean number of hours spent by parents on school work or school-related activities is different than 5 hours per week.