MATHEMATICS 201-510-LW
Business Statistics
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Fall 2008

XV – Estimating μ (small samples)
SOLUTIONS

1. Find the following

   a) $t_{(25,0.05)} = 1.708$ b) $t_{(10,0.10)} = 1.372$ c) $t_{(15,0.01)} = 2.602$ d) $t_{(21,0.025)} = 2.080$

   e) $t_{(21,0.95)} = -t_{(21,0.05)} = -1.721$ f) $t_{(26,0.975)} = -t_{(26,0.025)} = -2.056$ g) $t_{(27,0.99)} = -t_{(27,0.01)} = -2.473$ h) $t_{(60,0.025)} = 2.000$

2. A random sample of 15 statistics students showed that the mean time taken to solve an Excel assignment is 19 minutes with a standard deviation of 3 minutes. Construct a 99% confidence interval for the mean time taken by all statistics students to solve this Excel assignment. Assume that the time taken to solve this Excel assignment by all students follows a normal distribution.

   Step 1 Assumptions: The sampled population is normally distributed

   Step 2 a) Test statistic: t with $df = n-1 = 14$

   b) Level of confidence: $1-\alpha = 0.99$ or $\alpha = 0.01$

   Step 3 Point estimate: $\overline{x} = 19$ minutes

   Step 4 a) $t_{\left(df,\frac{\alpha}{2}\right)} = t_{(14,0.005)} = 2.977$

   b) $E = t_{\left(df,\frac{\alpha}{2}\right)} \frac{s}{\sqrt{n}} = 2.977 \frac{3}{\sqrt{15}} = 2.31$

   c)
   $$\overline{x} - E < \mu < \overline{x} + E$$
   $$19 - 2.31 < \mu < 19 + 2.31$$
   $$16.69 < \mu < 21.31$$

   Step 5 The 99% confidence interval for the mean time taken by all statistics students to solve this Excel assignment is 16.69 minutes to 21.31 minutes.

3. Six artichoke plants at a farm were selected at random. Here is the number of artichokes produced by each plant last year.

   32 17 51 40 36 34

   Construct a 99% confidence interval for the mean number of artichokes produced by all artichoke plants. Assume that the number of artichokes produced by all artichoke plants follows a normal distribution.

   Step 1 Assumptions: The sampled population is normally distributed

   Step 2 a) Test statistic: t with $df = n-1 = 5$

   b) Level of confidence: $1-\alpha = 0.99$ or $\alpha = 0.01$

   Step 3 Point estimate: $\overline{x} = 35$ artichokes

   Step 4 a) $t_{\left(df,\frac{\alpha}{2}\right)} = t_{(5,0.005)} = 4.032$

   b) $E = t_{\left(df,\frac{\alpha}{2}\right)} \frac{s}{\sqrt{n}} = 4.032 \frac{11.1}{\sqrt{6}} = 18.27$

   c) $\overline{x} - E < \mu < \overline{x} + E$

   $35 - 18.27 < \mu < 35 + 18.27$

   $16.73 < \mu < 53.27$

   Step 5 The 99% confidence interval for the mean number of artichokes produced by all artichoke plants is 16.73 to 53.27 artichokes.

4. The Dean of Registration at a College has implemented a new telephone registration system. The company that sold the system to the college claims that the mean length of phone calls is less than 10 minutes. To check this, the Dean randomly selected 20 calls, which had a mean length of 7.1 minutes and a standard deviation of 1.89 minutes. Assume that the length of phone calls is normally distributed.

   a) Construct a 95% confidence interval for the mean length of phone calls.

   Step 1 Assumptions: The sampled population is normally distributed

   Step 2 a) Test statistic: t with $df = n-1 = 19$

   b) Level of confidence: $1-\alpha = 0.95$ or $\alpha = 0.05$

   Step 3 Point estimate: $\overline{x} = 7.1$ minutes

   Step 4 a) $t_{\left(df,\frac{\alpha}{2}\right)} = t_{(19,0.025)} = 2.093$

   b) $E = t_{\left(df,\frac{\alpha}{2}\right)} \frac{s}{\sqrt{n}} = 2.093 \frac{1.89}{\sqrt{20}} = 0.88$

   c) $\overline{x} - E < \mu < \overline{x} + E$

   $7.1 - 0.88 < \mu < 7.1 + 0.88$

   $6.22 < \mu < 7.98$

   Step 5 The 95% confidence interval for the mean length of phone calls is 6.22 to 7.98 minutes.

   b) Is the president's claim consistent with your interval? Explain.

   Yes it is consistent, since all the values in the interval are less than 10.

5. Ten randomly selected shut-ins were each asked to list how many hours of television they watched per week. The results are

82	66	90	94	75	88	80	94	110	91

Determine the 90% confidence interval for the mean number of hours of television watched per week by shut-ins. Assume the number of hours is normally distributed.

Step 1 Assumptions: The sampled population is normally distributed

Step 2 a) Test statistic: t with $df = n - 1 = 9$

b) Level of confidence: $1-\alpha = 0.90$ or $\alpha = 0.10$

Step 3 Point estimate: $\overline{x} = 87.0$ hours

Step 4 a) $t_{\left(df, \frac{\alpha}{2}\right)} = t_{(9, 0.05)} = 1.833$

b) $E = t_{\left(df, \frac{\alpha}{2}\right)} \frac{s}{\sqrt{n}} = 1.883 \frac{12.07}{\sqrt{10}} = 7.00$

c) $\overline{x} - E < \mu < \overline{x} + E$

$87.0 - 7.00 < \mu < 87.0 + 7.00$

$80.00 < \mu < 94.00$

Step 5 The 90% confidence interval for the mean number of hours of television watched per week by shut-ins is 80.00 to 94:00 hours.

6. The pulse rate for 13 adult women were

83	58	70	56	76	64	80
76	70	97	68	78	108	

Find a 90% confidence interval for the mean pulse rate of adult women. Assume that the pulse rate for adult women is normally distributed.

Step 1 Assumptions: The sampled population is normally distributed

Step 2 a) Test statistic: t with $df = n - 1 = 9$

b) Level of confidence: $1-\alpha = 0.90$ or $\alpha = 0.10$

Step 3 Point estimate: $\overline{x} = 75.7$

Step 4 a) $t_{\left(df, \frac{\alpha}{2}\right)} = t_{(12, 0.05)} = 1.782$

b) $E = t_{\left(df, \frac{\alpha}{2}\right)} \frac{s}{\sqrt{n}} = 1.782 \frac{14.54}{\sqrt{13}} = 7.19$

c) $\overline{x} - E < \mu < \overline{x} + E$

$75.7 - 7.19 < \mu < 75.7 + 7.19$

$68.51 < \mu < 82.89$

Step 5 The 90% confidence interval for the mean pulse rate of women is 68.51 to 82.89 hours.