

MATHEMATICS 201-510-LW

Business Statistics

Martin Huard

Fall 2008

XIII – Statistical Process Control

- Suppose the statistics $\bar{\bar{x}} = 143.1$ and $\bar{R} = 17.5$ are obtained from 50 samples of size 9.
 - What is the centerline and the control limits for an \bar{x} chart?
 - What is the centerline and the control limits for an R chart?
- If, when a production process is under control, it has been determined that $\mu = 76.31$ and $\sigma = 3.07$, what is the centerline and the control limits, if the process is monitored by taking samples of size 8 for
 - an \bar{x} chart?
 - an σ chart?
- In order to ensure that a manufacturing process is under control, 50 samples of size 100 are taken and the number of defectives in each sample is counted. If the mean sample proportion is 0.021, what is the center line and the control limits for a p chart?
- In an attempt to monitor customer satisfaction, an air line recorded the number of complaints received per day for the last 50 days. If in the past the average number of complaints was 7 per day, what is the center line and the control limits for a c chart?
- Each day the quality control department at a candy factory takes a random sample of 5 chocolate bars and measures their weight, to see if the process is in control. Here are the results for the past 12 days.

Day						\bar{x}	R
1	63.8	65.8	65.2	64.6	65.8	65.04	2.0
2	64.8	64.3	65.4	64	64.7		1.4
3	64.1	65.6	64.2	63.5	65.1	64.50	2.1
4	64.6	65.8	65.9	63.6	65.7	65.12	2.3
5	64.7	66.4	65.5	64.7	65.4	65.34	
6	65.7	64.6	64.4	66.1	64.8	65.12	1.7
7	66.3	65.1	66.0	64.2	65.7	65.46	2.1
8	65.4	66.2	64.9	61.4	65.6	64.70	4.8
9	66.7	66.0	65.2	65.9	65.6	65.88	1.5
10	62.4	67.0	66.4	66.2	68.7	66.14	6.3
11	66.6	62.9	66.4	66.2	67.8	65.98	4.9
12	68.3	65.7	67.5	66.3	62.3	66.02	6.0

- Fill in the missing numbers in the last two columns.
- Make an \bar{x} chart. Is the process in control?
- Make an R chart. Is the process in control?

6. A coffee machine fills a cup of coffee to a fixed volume. When the machine is in control, the mean volume is 250 ml with a standard deviation of 3.1 ml. To see whether the machine is in control, samples of 4 cups were taken each day for the past 20 days and their volume measured. Here are the results.

Day					\bar{x}	R
1	251.7	243.1	248.0	248.2	247.75	8.6
2	248.9	245.7	252.7	242.2	247.38	10.5
3	253.8	250.4	242.3	249.5	249.00	11.5
4	252.5	247.6	249.8	256.3	251.55	8.7
5	251.7	244.1	250.9	247.7	248.60	7.6
6	248.7	248.4	252.9	251.5	250.38	4.5
7	248.2	250.3	247.5	251.0	249.25	3.5
8	245.7	252.0	249.6	252.9	250.05	7.2
9	257.6	260.9	256.9	254.8	257.55	6.1
10	264.7	254.0	259.4	257.3	258.85	10.7
11	259.1	256.6	257.2	261.4	258.58	4.8
12	254.6	260.5	255.5	262.0	258.15	7.4
13	255.0	254.8	267.4	254.7	257.98	12.7
14	262.6	258.9	260.1	255.9	259.38	6.7
15	255.2	253.6	260.2	260.4	257.35	6.8
16	257.8	262.2	257.3	259.9	259.30	4.9
17	261.9	260.5	255.4	257.9	258.93	6.5
18	255.1	259.3	258.0	260.0	258.10	4.9
19	255.9	251.6	257.2	254.1	254.70	5.6
20	256.0	253.6	259.8	253.0	255.60	6.8

- a) Make an \bar{x} chart using the known μ and σ . Is the process in control?
 b) Make an σ chart using the known μ and σ . Is the process in control?
7. The Canadian Math Delivery Service (CMDS) guarantees delivery of small packages by 9:00 AM the next working day anywhere in Canada. To monitor the efficiency of their system, CMDS took random samples of 200 packages during the last 15 working days and observed the following number of packages which were delivered after the deadline. Is the process in control?
- 2 3 4 3 1 6 9 2 7 19 5 6 8 10 6
8. The number of “near misses” recorded for the past 20 months at Zola airport are 5, 3, 1, 2, 4, 1, 2, 3, 0, 1, 3, 3, 2, 4, 3, 5, 3, 7, 3, and 7. Is the process in control?

9. A company that produces CD's for record company has been receiving complaints that a significant number of CD's are defective. To remedy the problem, the company decided to take random samples of 250 CD every hour and check them to see if they are defective or not. Here are the results from the first 40 hours. Is the process out of control?

6 13 17 17 10 16 16 11 5 6 10 14 14 5 11 11 14 15 11 10
 11 12 7 15 9 7 10 3 14 12 17 9 14 16 7 20 13 18 11 10

10. The following data represents the number fires per week that were signaled to the Quebec City Fire Department for the past 25 weeks. Was the number of fires out of control at any time during the last 25 weeks?

9 4 13 10 5 10 13 2 10 21 15 10 16 9 11 12 9 15 7 12 10 14 9 7 6

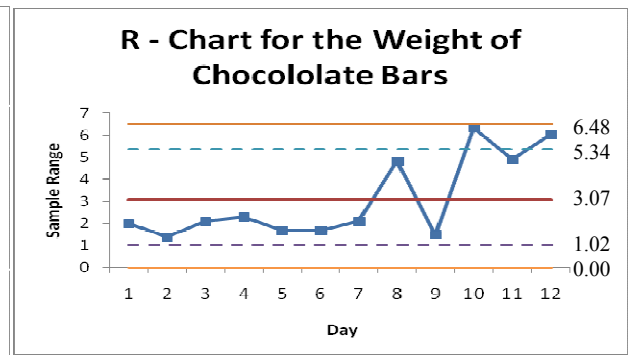
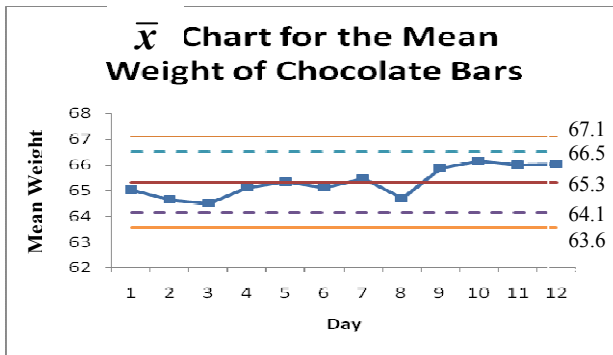
11. A new industrial oven has been installed at Great Bakery. In order to test the distribution of the temperature inside the oven, an employee read the temperature in 7 different places inside the oven every 15 minutes for the first four hours. Here are the results (in degrees celcius).

Time								\bar{x}	R
8:00 A.M.	396	443	418	395	385	396	382	402.1	61
8:15 A.M.	394	389	401	378	405	409	404	397.1	31
8:30 A.M.	409	400	411	408	414	409	408	408.4	14
8:45 A.M.	401	407	386	415	363	400	384		52
9:00 A.M.	375	412	377	398	376	395	391	389.1	37
9:15 A.M.	407	401	373	418	395	381	437	401.7	64
9:30 A.M.	406	403	368	394	366	430	418	397.9	
9:45 A.M.	381	423	397	390	407	414	397	401.3	42
10:00 A.M.	396	397	397	396	396	397	395		2
10:15 A.M.	387	403	363	395	403	382	386	388.4	40
10:30 A.M.	408	399	385	413	395	393	390	397.6	28
10:45 A.M.	399	400	404	388	417	379	416	400.4	38
11:00 A.M.	387	416	371	414	418	420	377	400.4	49
11:15 A.M.	421	423	416	401	413	411	411	413.7	22
11:30 A.M.	409	404	401	389	405	407	396	401.6	
11:45 A.M.	388	417	391	395	395	397	435	402.6	47

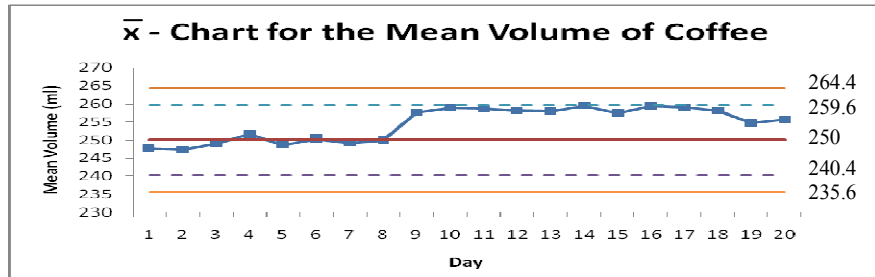
- a) Fill in the missing information in the table.
 b) Make an \bar{x} chart. Is the process in control?
 c) Make an R chart. Is the process in control?

ANSWERS

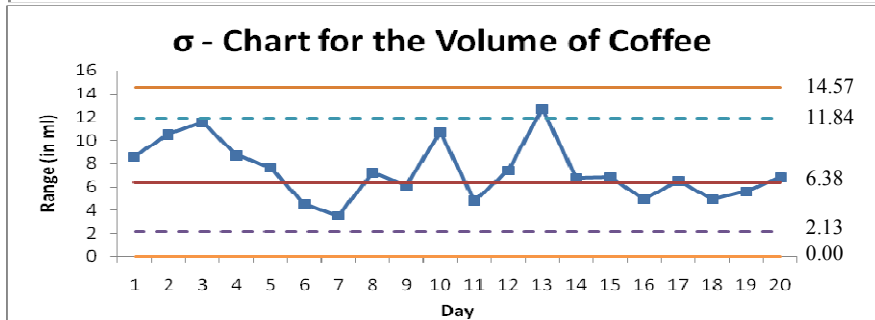
1. a) Centerline: 143.1 UCL: 149.0 $\frac{2}{3}$ UCL : 147.0 $\frac{2}{3}$ LCL : 139.2 LCL 137.2
 b) Centerline: 17.5 UCL: 31.78 $\frac{2}{3}$ UCL : 27.02 $\frac{2}{3}$ LCL : 7.98 LCL 3.22
2. a) Centerline: 76.31 UCL: 86.31 $\frac{2}{3}$ UCL : 82.97 $\frac{2}{3}$ LCL : 69.65 LCL 66.31
 b) Centerline: 8.74 UCL: 16.29 $\frac{2}{3}$ UCL : 13.78 $\frac{2}{3}$ LCL : 3.71 LCL 1.19
3. Centerline: 0.021 UCL: 0.0640 $\frac{2}{3}$ UCL : 0.0497 $\frac{2}{3}$ LCL : 0 LCL 0
4. Centerline: 7 UCL: 14.94 $\frac{2}{3}$ UCL : 12.29 $\frac{2}{3}$ LCL : 1.71 LCL 0
5. a) $\bar{x}_2 = 64.64$ $R_5 = 1.7$
 b) $\bar{\bar{x}} = 65.33$ grams In control
 c) $\bar{\bar{R}} = 3.07$ grams Out of control (signal 3)



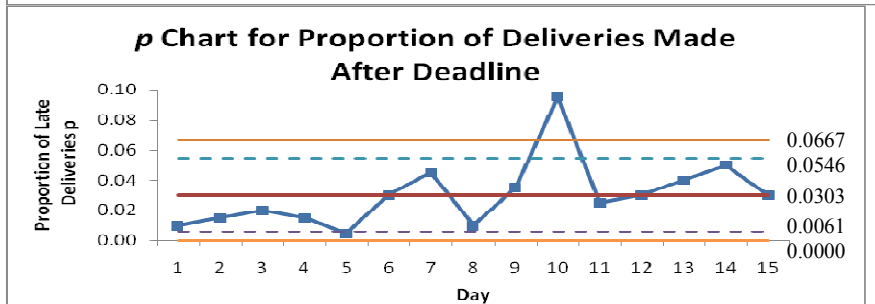
6. a) Out of control
Signal 2



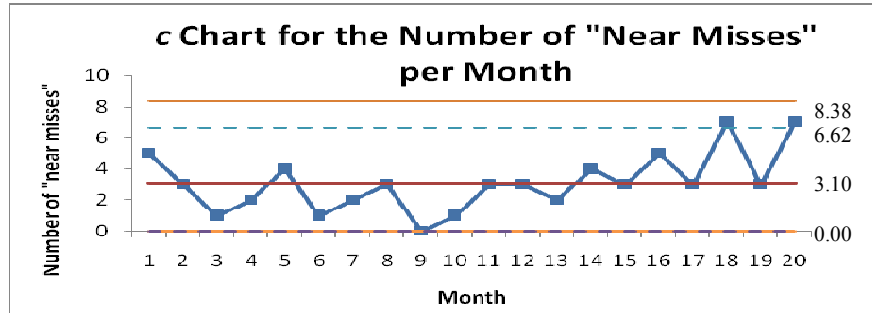
- b) In control



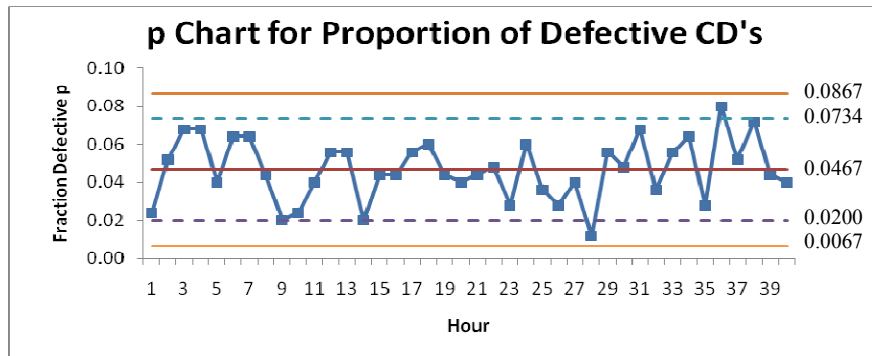
7. Out of Control
signal 1



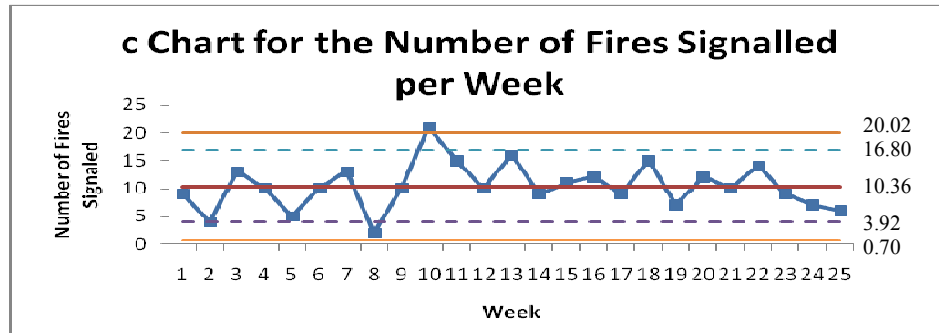
8. Out of Control
Signal 3



9. In control



10. Out of Control
Signal 1



11. a) $\bar{x}_4 = 393.7$, $\bar{x}_9 = 396.3$, $R_7 = 64$, $R_{15} = 20$

b) In Control

c) Out of Control Signal 1 and 3
(although only signal 3 is of concern)

