MATHEMATICS 201-510-LW
Business Statistics
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XII – Sampling Distributions

1. A population consists of the values 3, 4, 9 and 16.
   a) List 16 possible samples of size 2 with replacement.
   b) For each sample, find the mean, median, range, variance and standard deviation. Give the probability distribution for each of these statistics.
   c) For each statistic in (b), find the mean of the results.
   d) Find the mean, median, range, variance and standard deviation for the population and compare the answer to the one obtained in (c), and determine whether each sample statistic targets the value of the population parameter.
   e) For each sample, find the proportion of odd numbers. Give the probability distribution for this statistic. Find the mean for this statistic. What is the population proportion? Does the sample statistic target the population parameter?

2. Consider the population in example 1.
   a) Find the variance of the sample means. How does it compare to the population variance?
   b) Find the variance of the sample proportions. Does the result satisfy $\sigma_{\hat{p}}^2 = npq$?

3. The speeds of all cars traveling on a stretch of Highway 20 are normally distributed with a mean of 110 kilometers per hour and a standard deviation of 5 kilometers per hour. Calculate the mean speed and standard deviation for a sample of 20 cars.

4. A study has shown that the mean weekly wage for manufacturing workers was $517. Assume that the weekly wages for all manufacturing workers are normal distributed with a mean of $517 and a standard deviation of $65. Find the probability that the mean weekly wage for a random sample of 25 manufacturing workers taken from this population would be
   a) between $527 and $537
   b) within 15$ of the population mean weekly wage.
   c) Lower that the population mean weekly wage by $20 or more

5. The ages of all university students follow a distribution with a mean of 24 years and a standard deviation of 4 years. Find the probability that the mean age for a random sample of 36 students would be
   a) between 20 and 25 years
   b) less than 23 years
   c) more than 26

6. It is widely know that people in England drink a lot of tea – on average more than 3 cups per day. Suppose that the mean number of cups of tea that people living in England drink per day is 3.4 cups, with a standard deviation of 1.3 cups per day.
   a) Find the probability that a sample of 144 people produces a mean of more than 3.65 cups per day.
   b) If the sample mean was 3.65 cups per day, what would we be led to conclude?

7. A manufacturer of light bulbs says that its light bulbs have a mean life of 700 hr and a standard deviation of 120 hr. You purchased 144 of these bulbs with the idea that you would purchase more if the mean life of you sample were more than 680 hr. What is the probability that you will not buy again from this manufacturer?

8. A trucking firm delivers crates of appliances for a large retail operation. The weight of the crates is normally distributed with a mean weight of 150 kg and standard deviation of 20 kg.
   a) If a truck can carry 2000 kg and 25 crates need to be picked up, what is the probability that the 25 crates will have an aggregate weight greater that the truck's capacity?
   b) If the truck has a capacity of 4000 kg, what is the probability that it will be able to carry the entire lot of 25 crates?

9. At a Pepsi bottling plant, soda is bottled into plastic containers that are suppose to contain 2 liters. The actual volume of Pepsi is a random variable with a mean of 2.008 liters and a standard deviation of 0.018 liters. A random sample of forty-eight 2 liter bottles is taken.
   a) What is the probability that the sample mean is lower than 2 liters?
   b) Within what limits would the middle 95% of the sampling distribution of sample means fall?

10. A food processing plant packages butter in blocks that are suppose to contain 454 grams (one pound). The quality control department regularly takes samples of 40 blocks and finds the mean weight to see if it is 454 grams or above. If the standard deviation on the packaging machine is calibrated to be 3.1 grams, what should be the weight of butter the machine will package so that the probability that quality control finds the mean weight of a sample to be less than 454 grams is less than 2%?

ANSWERS

1. a) Samples: $(3,3),(3,4),(3,9),(3,16)$, $(4,3),(4,4),(4,9),(4,16)$, $(9,3),(9,4),(9,9),(9,16)$, $(16,3),(16,4),(16,9),(16,16)$

Sampling Distribution of Sample Means

$\bar{x}$	$p(\bar{x})$
3	1/16
3.5	1/8
4	1/16
6	1/8
6.5	1/8
9	1/16
9.5	1/8
10	1/8
12.5	1/8
16	1/16

Sampling Distribution of Sample Medians

Me	$p(Me)$
3	1/16
3.5	1/8
4	1/16
6	1/8
6.5	1/8
9	1/16
9.5	1/8
10	1/8
12.5	1/8
16	1/16

Sampling Distribution of Sample Range

R	$p(R)$
0	1/4
1	1/8
5	1/8
6	1/8
7	1/8
12	1/8
13	1/8

Sampling Distribution of Sample Variance

s^2	$p(s^2)$
0	1/4
0.5	1/8
12.5	1/8
18	1/8
24.5	1/8
72	1/8
84.5	1/8

Sampling Distribution of Sample Proportion of Odd Numbers

$\hat{p}$	$p(\hat{p})$
0	1/4
0.5	1/8
1	1/4

Sampling Distribution of Sample Standard Deviation

s	$p(s)$
0	1/4
0.7071	1/8
3.5355	1/8
4.2426	1/8
4.9497	1/8
8.4853	1/8
9.1924	1/8

c) $\mu_{\bar{x}} = 8$, $\mu_{Me} = 8$, $\mu_R = 5.5$, $\mu_{s^2} = 26.5$, $\mu_s = 3.8891$, $\mu_{\hat{p}} = \frac{1}{2}$

d) $\mu = 8$, $Me = 6.5$, $R = 13$, $\sigma^2 = 26.5$, $\sigma = 5.1478$, $p = \frac{1}{2}$

2. a) $\sigma_{\bar{x}}^2 = 13.25 = \frac{\sigma^2}{2}$ b) $\sigma_{\hat{p}}^2 = \frac{1}{8} = \frac{p(1-p)}{2}$
3. $\bar{x} = 110$ km/h $\sigma_{\bar{x}} = 1.12$ km/h
4. a) 0.1588 b) 0.7498 c) 0.0618
5. a) 0.9332 b) 0.0668 c) 0.0013
6. a) 0.0104
   b) Since the chances of obtaining a sample that is at least this extreme is very small, there is a strong possibility that either the sample is not representative of the population, or the given population mean and standard deviation are wrong.
7. 0.0228
8. a) 1.000 b) 0.9938
9. a) 0.0010 b) 2.0029 to 2.0131
10. 455 grams