Answer all questions and show all your work.

Question 1 (10 points)
A systems specialist has studied the workflow of clerks all doing the same inventory work. Based on this study, she designed a new work flow layout for the inventory system. To compare average production for the old and new methods, a random sample of five clerks was used. The average production rate (number of inventory items processed per shift) for each clerk was measured both before and after the new system was introduced. The results are show below. Test the claim that the new system increases the mean number of items processed per shift. Use a 5 % level of significance. Use the p-value approach. Assume that the production rates are normally distributed.

<table>
<thead>
<tr>
<th>Old</th>
<th>New</th>
</tr>
</thead>
<tbody>
<tr>
<td>116</td>
<td>123</td>
</tr>
<tr>
<td>108</td>
<td>114</td>
</tr>
<tr>
<td>93</td>
<td>112</td>
</tr>
<tr>
<td>88</td>
<td>82</td>
</tr>
<tr>
<td>119</td>
<td>127</td>
</tr>
<tr>
<td>$d = N - O$</td>
<td>7</td>
</tr>
</tbody>
</table>

Step 1  Assumptions: $X_O \sim N(\mu_O, \sigma_O^2)$
        $X_N \sim N(\mu_N, \sigma_N^2)$

Test statistic: $t \sim t(4)$

Step 2  $H_0: \mu_d = 0$
        $H_A: \mu_d > 0$

Step 3  Right-tailed test with $\alpha = 0.05$
        $t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} = \frac{6.8 - 0}{\frac{8.871}{\sqrt{5}}} = 1.71$
        $0.073 < p-value < 0.082$

Step 4  a) $p$-value $> 0.073$ $> \alpha = 0.05$
        b) Fail to reject $H_0$.

\[ \therefore \] There is not sufficient evidence at the 5% level of significance to conclude that the system increases the mean number of items processed per shift.
**Question 2** (10 points)
Does life insurance matter if you are male or female? The following is based on information from *Consumer Reports*. For similar benefits (male and female), the annual premiums paid by a person 45 years old for a $250 000 annual renewable term life insurance policy were as follows:

- **Males**: In a sample of 32 males, the average annual premium was $483.43 with standard deviation $126.62.
- **Females**: In a sample of 41 females, the average annual premium was $414.43 with standard deviation $105.99.

At the 5% level of significance, test the claim that the annual premiums between male and female are different. Use the classical approach.

**Step 1** Assumptions: The samples are independent
\[
\sigma_M^2 = \sigma_W^2 = \sigma^2
\]
\[
n_M = 32 \geq 30 \quad \therefore \text{by CLT, } \bar{X}_M \sim N\left(\mu_M, \frac{\sigma^2}{n_M}\right)
\]
\[
n_W = 41 \geq 30 \quad \therefore \text{by CLT, } \bar{X}_W \sim N\left(\mu_W, \frac{\sigma^2}{n_W}\right)
\]

Test statistic: \( t \sim t(71) \)
\( \alpha = 0.05 \)

**Step 2**
- \( H_0 : \mu_M - \mu_W = 0 \)
- \( H_1 : \mu_M - \mu_W \neq 0 \)

**Step 3**
\[
t_{(df, \alpha / 2)} = t_{(71,0.025)} = 2.000
\]
\[
s_p = \sqrt{\frac{(n_L - 1)s_L^2 + (n_F - 1)s_F^2}{n_L + n_F - 2}} = \sqrt{\frac{31 \cdot 126.62^2 + 40 \cdot 105.99^2}{71}} = 115.45
\]
\[
t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{483.43 - 414.43}{115.45 \sqrt{\frac{1}{32} + \frac{1}{41}}} = 2.534
\]

**Step 4** \( t \) is in the critical region
Reject \( H_0 \).
\( \therefore \) There is sufficient evidence, at the 5% level of confidence, to conclude that the annual premiums between male and female are different.
Question 3 (10 points)
Laura, the owner of a sushi franchise, wants to determine if more women like sushi than men. In a random sample of 320 women, 200 said they liked sushi, and in a random sample of 392 men, 210 said they liked sushi. Construct a 98% confidence interval for the difference in the proportion of men and women who like sushi.

Step 1 Assumptions: \( n_w \hat{p}_w = 200 > 5 \)  \( n_w \hat{q}_w = 120 > 5 \)
\( n_m \hat{p}_m = 210 > 5 \)  \( n_m \hat{q}_m = 182 > 5 \)
Samples are independent
Test statistic \( z \sim N(0,1) \)
\( 1 - \alpha = 0.98 \)
Step 2 Point estimate: \( \hat{p}_w - \hat{p}_m = \frac{200}{320} - \frac{210}{392} = 0.0893 \)
Step 3 \( z_{\frac{\alpha}{2}} = z_{0.01} = 2.33 \)
\[ E = z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_w \hat{q}_w}{n_w} + \frac{\hat{p}_m \hat{q}_m}{n_m}} = 2.33 \sqrt{\frac{200}{320} \frac{120}{320} + \frac{210}{392} \frac{182}{392}} = 0.0861 \]
\( (\hat{p}_w - \hat{p}_m) - E < p_w - p_m < (\hat{p}_w - \hat{p}_m) + E \)
\( 0.0893 - 0.0861 < p_w - p_m < 0.0893 + 0.0861 \)
\( 0.0031 < p_w - p_m < 0.1754 \)
Step 4 The 98% confidence interval for the difference in the proportion of men and women who like sushi is 0.3% to 17.5%.
**Question 4** (10 points)

A random sample of 100 CEGEP students was selected where each was asked for his mother tongue and whether they are planning to go to a French or an English university. Can you conclude, at the 5% level of significance, the university a CEGEP student goes to is not independent of his mother tongue? Use the classical approach.

<table>
<thead>
<tr>
<th>Mother Tongue</th>
<th>Language of University</th>
<th>French</th>
<th>English</th>
</tr>
</thead>
<tbody>
<tr>
<td>French</td>
<td></td>
<td>40</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(30)</td>
<td>(20)</td>
</tr>
<tr>
<td>English</td>
<td></td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(18)</td>
<td>(12)</td>
</tr>
<tr>
<td>Other</td>
<td></td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(12)</td>
<td>(8)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>60</td>
<td>40</td>
</tr>
</tbody>
</table>

**Step 1**
Assumptions: The classes are all inclusive and mutually exclusive

Test statistic: $\chi^2 \sim \chi^2(2)$

$\alpha = 0.05$

**Step 2**

$H_0$: The university a CEGEP student goes to is independent of his mother tongue.

$H_A$: The university a CEGEP student goes to is not independent of his mother tongue.

**Step 3**

Right-tailed test

$\chi^2_{(df, \alpha)} = \chi^2_{(2, 0.05)} = 5.99$

$\chi^2 = \sum \frac{(O - E)^2}{E}$

$\chi^2 = \frac{(40 - 30)^2}{30} + \frac{(10 - 18)^2}{18} + \frac{(10 - 12)^2}{12} + \frac{(10 - 20)^2}{20} + \frac{(20 - 12)^2}{12} + \frac{(10 - 8)^2}{8}$

$\chi^2 = 18.06$

**Step 4**

a) $\chi^2$ is in the critical region

b) Reject $H_0$.

: There is sufficient evidence at the 5% level of significance to conclude that the university a CEGEP student goes to is dependent of his mother tongue.
Question 5 (10 points)
The owner of a physiotherapy clinic is studying the sometimes large spread in waiting time for patients to obtain an appointment for consultation. In a random sample of 25 patients, the standard deviation for the waiting times was 3.4 days. Assuming that the waiting times are normally distributed, find a 95% confidence interval for the standard deviation of the waiting time for patients to obtain an appointment for consultation.

Step 1  Assumptions: \( X \sim N(\mu, \sigma^2) \) thus \( \chi^2 = \frac{(n-1)s^2}{\sigma^2} \sim \chi^2 (24) \)

Level of confidence: \( 1 - \alpha = 0.95 \) or \( \alpha = 0.05 \)

Step 2  Point Estimate: \( s = 3.4 \) days 

\[ s^2 = 3.4^2 = 11.56 \text{ days}^2 \]

Step 3  

a) \( \chi^2_{(24, 0.025)} = \chi^2_{(24, 0.975)} = 12.40 \)

\[ \chi^2_{(24, 0.975)} = \chi^2_{(24, 0.025)} = 39.36 \]

b) \( \frac{24 \cdot 3.4^2}{\chi^2_{(24, 0.975)}} < \sigma^2 < \frac{24 \cdot 3.4^2}{\chi^2_{(24, 0.025)}} \)

\[ \frac{24 \cdot 3.4^2}{39.36} < \sigma^2 < \frac{24 \cdot 3.4^2}{12.40} \]

\[ 7.05 < \sigma^2 < 22.37 \]

\[ 2.65 < \sigma < 4.73 \]

Step 4  The 95% confidence interval for the standard deviation of waiting times is 2.65 days to 4.73 days.