

## MATHEMATICS 201-510-LW

Business Statistics

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Fall 2008

### Assignment #3 SOLUTIONS

This assignment is due on **Friday October 31, 2008** at the beginning of class.  
Complete solutions are expected.

#### Question 1 (12 points)

A taxi company wanted to get an idea on the average distance of a taxi ride to see if the price charged per kilometer is adequate. For this, a random sample of 33 taxi rides was taken, and the distance (in kilometers) of the ride noted. Here are the results.

11.6	5.5	5.7	5.1	3.7	6.1	7.6	2.0	5.9	1.3	11.3
6.2	11.0	12.3	5.7	2.0	0.8	3.8	9.1	1.3	2.6	13.4
6.6	8.0	12.0	4.5	6.7	5.3	0.2	6.2	1.5	7.7	9.9

- a) Construct a 97% interval for the mean distance traveled.

Step 1 Assumptions:  $n = 33 \geq 30$

Step 2 a) Test statistic:  $z$

b) Level of confidence:  $1 - \alpha = 0.97$  or  $\alpha = 0.03$

Step 3 Point estimate:  $\bar{x} = 6.139$  km

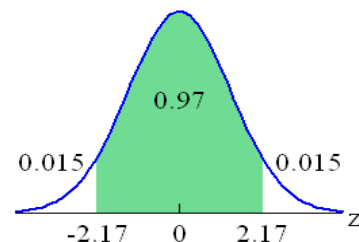
Step 4 a)  $z_{\frac{\alpha}{2}} = z_{0.015} = 2.17$

b)  $E = z_{0.015} \frac{s}{\sqrt{n}} = 2.17 \frac{3.675}{\sqrt{33}} = 1.388$

c)  $\bar{x} - E < \mu < \bar{x} + E$

$$6.139 - 1.388 < \mu < 6.139 + 1.388$$

$$4.751 < \mu < 7.527$$



Step 5 The 97% confidence interval for the mean distance of a taxi ride is 4.75 to 7.53 km.

- b) Can you conclude, at the 5% level of significance, that the mean distance on a taxi ride is more than 5 km? Use the classical approach.

Step 1 Assumptions:  $n = 33 \geq 30$

Step 2  $H_0: \mu = 5$  km

$H_a: \mu > 5$  km

Step 3 a) Test statistic:  $z$

b) Right-tailed test with  $\alpha = 0.05$

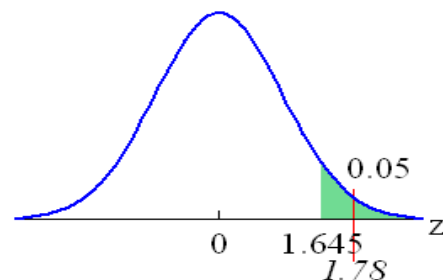
c)  $z_{\alpha} = z_{0.05} = 1.645$

Step 4  $z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{6.139 - 5}{\frac{3.675}{\sqrt{33}}} = 1.78$

Step 5 a)  $z$  is in the critical region

b) Reject  $H_0$ .

$\therefore$  There is sufficient evidence at the 5% level of significance to conclude that the mean distance of a taxi ride is more than 5 km.



**Question 2** (12 points)

An insurance company wants to estimate the average amount of a claim. A random sample of 16 claims was taken. Here are the results (in \$).

664	714	803	851	784	947	845	845
847	886	804	725	812	929	738	833

Assume that the amount of a claim is normally distributed.

- a) Construct a 90% interval for the mean value of a claim.

Step 1 Assumptions: The sampled population is normally distributed

Step 2 a) Test statistic:  $t$  with  $df = n - 1 = 15$

b) Level of confidence:  $1 - \alpha = 0.99$  or  $\alpha = 0.01$

Step 3 Point estimate:  $\bar{x} = 814.19$  \$

Step 4 a)  $t_{(df, \frac{\alpha}{2})} = t_{(15, 0.05)} = 1.753$

b)  $E = t_{(df, \frac{\alpha}{2})} \frac{s}{\sqrt{n}} = 1.753 \frac{76.66}{\sqrt{16}} = 33.60$

a)  $\bar{x} - E < \mu < \bar{x} + E$

$$814.19 - 35.60 < \mu < 814.19 + 35.60$$

$$780.59 < \mu < 847.79$$

Step 5 The 90% confidence interval for a mean claim is \$780.59 to \$847.79.

- b) Can you conclude that the mean value of a claim is different than \$865? Use the  $p$ -value approach with a 1% level of significance.

Step 1 Assumptions: Population is normally distributed

Step 2  $H_0 : \mu = \$865$

$H_a : \mu \neq \$865$

Step 3 a) Test statistic:  $t$  with  $df = n - 1 = 15$

b) Two-tailed test with  $\alpha = 0.01$

Step 4 a)  $t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{814.19 - 865}{\frac{77.66}{\sqrt{16}}} = -2.65$

b)  $2 \cdot 0.008 < p\text{-value} < 2 \cdot 0.01$   
 $0.016 < p\text{-value} < 0.02$

Step 5 a)  $p\text{-value} > 0.016 > \alpha = 0.01$

b) Fail to reject  $H_0$ .

$\therefore$  There is not sufficient evidence at the 1% level of significance to conclude that the mean value of a claim is different than \$865.

**Question 3** (7 points)

A random sample of Canadians was taken by an airline company, where each was asked if they plan to go on a vacation during the Christmas Holidays. Here are the results.

No	Yes	No	No	No	No	Yes	No
No	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Yes	No	No	No	No	No	No	No
No	Yes	No	No	No	Yes	No	Yes
No	No	No	No	Yes	No	No	No
Yes	Yes	Yes	No	No	No	No	No
Yes	Yes	Yes	No	Yes	Yes	No	No
No	No	Yes	Yes	No	No	Yes	No
No	No	No	Yes	No	No	No	No
No	Yes	Yes	No	Yes	Yes	No	No
No	No	No	No	No	No	No	Yes
Yes	No	No	No	No	Yes	No	No

Construct a 92% confidence interval for the proportion of Canadians who plan to go on a vacation during the Christmas Holidays.

Step 1 Assumptions:  $n = 96 > 20$

$$n\hat{p} = 33 > 5$$

$$n\hat{q} = 63 > 5$$

Step 2 a) Test statistic:  $z$

b) Level of confidence:  $1 - \alpha = 0.92$  or  $\alpha = 0.08$

Step 3 Point estimate:  $\hat{p} = \frac{r}{n} = \frac{33}{96} = 0.3438$

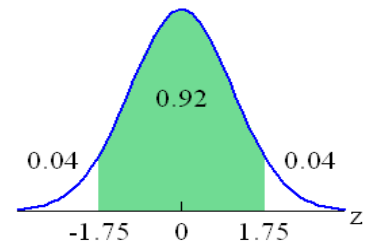
Step 4 a)  $z_{\frac{\alpha}{2}} = z_{0.04} = 1.75$

$$b) E = z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.75 \sqrt{\frac{33 \cdot 63}{96 \cdot 96}} = 0.0848$$

$$c) \bar{x} - E < p < \bar{x} + E$$

$$0.3438 - 0.0848 < p < 0.3438 + 0.0848$$


$$0.2590 < p < 0.4286$$



Step 5 The 92% confidence interval for the proportion of Canadians who would plan to go on a vacation during the Christmas Holidays is 25.9% to 42.9% .

Questions 4 to 8 are to be done using Excel.

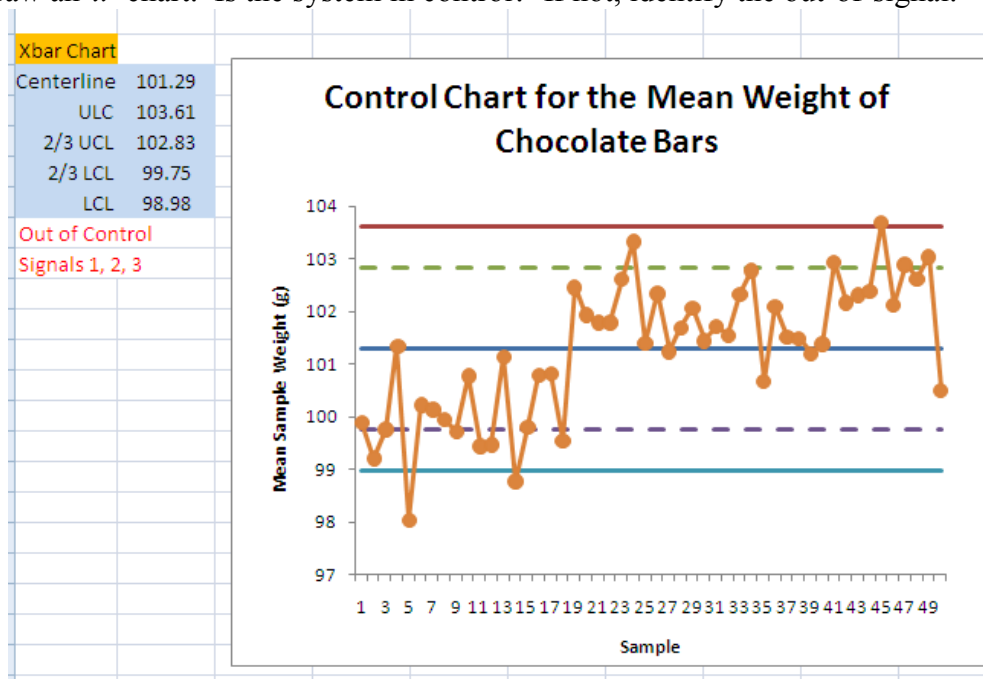
For these questions, hand-in the printouts of your Excel sheets and copy your Excel work in the Test folder for Business Statistics (W:\Tests\mhuard\Business Statistics\Assignment 3), where your name should be included in the name of the file (for example: Assignment 3 – Your Name). Make sure that your answers are well organized with appropriate labels, and rounded off to an appropriate number of decimal places.

Open the file “Data – Assignment 3” from my web site, and save it under “Assignment 3 – Your Name”. Note that you may have to enable macros to be able to generate the data. If the macros are not enabled (that is, if the data does not appear at the click of the button) then go to  - EXCEL OPTIONS – TRUST CENTER – TRUST CENTER SETTINGS – MACRO SETTINGS and choose the ENABLE ALL MACROS option. Note that you may need to close your document and open it again.

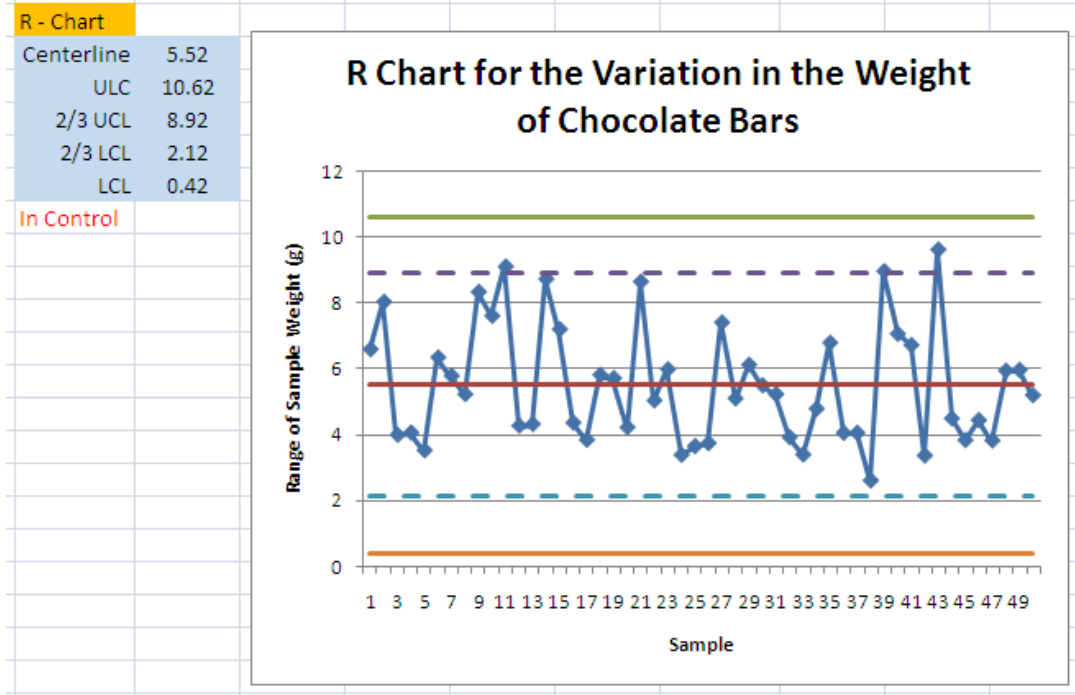
**Question 4** (8 points)

In order to monitor the production of ChocoMath chocolate bars, the quality control department took samples of 7 chocolate bars and measured their weight for the past 50 days.

- a) Go to the worksheet “Sheet1”, rename it appropriately, make the usual heading in cells A1:A4, then click on the “GENERATE DATA” button to get your data.
- b) Draw an  $\bar{x}$  chart. Is the system in control? If not, identify the out-of-signal.



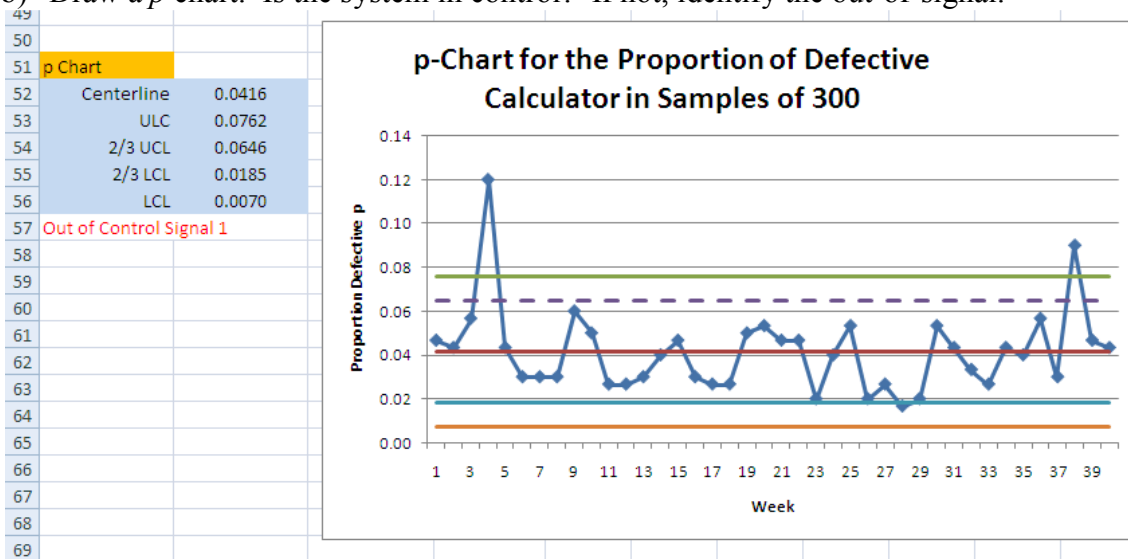
c) Draw an  $R$  chart. Is the system in control? If not, identify the out-of-signal.



**Question 5** (4 points)

The quality control department of a calculator manufacturer takes random samples of 300 calculators and notes the number of defective ones the sample contains. Samples were taken for the last 40 weeks.

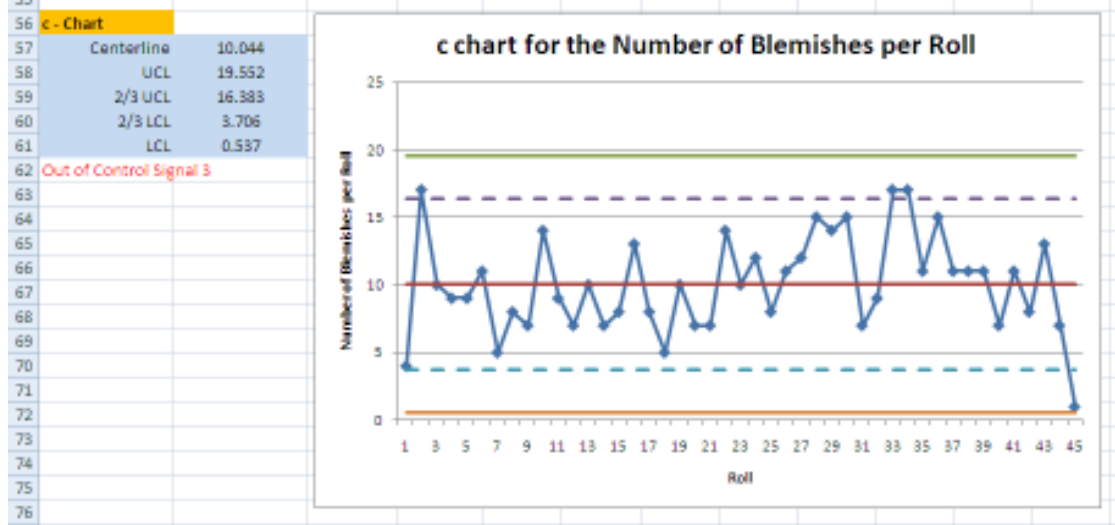
- Go to the worksheet “Sheet2”, rename it appropriately, make the usual heading in cells A1:A4, then click on the “GENERATE DATA” button to get your data.
- Draw a  $p$  chart. Is the system in control? If not, identify the out-of-signal.



**Question 6** (4 points)

A pulp and paper mill produces large rolls of paper that are sold to printing presses. In an effort to monitor the quality of their product, the number of blemishes (or imperfections) per roll was noted for the last 45 rolls.

- Go to the worksheet “Sheet3”, rename it appropriately, make the usual heading in cells A1:A4, then click on the “GENERATE DATA” button to get your data.
- Draw a *c* chart. Is the system in control? If not, identify the out-of-signal.



**Question 7** (6 points)

A researcher wants to estimate the mean weekly grocery bill of households. For this, a random sample of households was taken, where each gave the weekly grocery bill.

- Go to the worksheet “Sheet4”, rename it appropriately, make the usual heading in cells A1:A4, then click on the “GENERATE DATA” button to get your data.
- Construct a 94% confidence interval for the mean weekly grocery bill of households.

7						
8	<b>Bill (\$)</b>	<i>Bill (\$)</i>		<b>Step 1</b>	Assumptions: n =	97 ≥ 30
9	171.23			<b>Step 2</b>	Test statistic: z	
10	93.3	Mean	139.8295	<b>Step 3</b>	1 - α =	94%
11	120.45	Standard Error	3.190935	<b>Step 4</b>	Mean =	\$139.83
12	116.82	Median	143.27	<b>Step 5</b>	E =	6.00
13	158.78	Mode	#N/A	The 95% confidence interval for the mean grocery bill of households is		
14	137.52	Standard Deviation	31.42707		\$133.83	to \$145.83
15	110.01	Sample Variance	987.6604			
16	100.36	Kurtosis	-0.19726			
17	94.51	Skewness	-0.15645			
18	145.52	Range	154.71			
19	166.21	Minimum	57.03			
20	124.76	Maximum	211.74			
21	113.81	Sum	13563.46			
22	79.62	Count	97			
23	113.17	Confidence Level(94.0%)	6.07324			
24	152.11					

**Question 8** (7 points)

A car dealership claims that the annual mileage of Canadians is less than 20 000 km. To verify this claim, a random sample of Canadians was taken where each gave their mileage of the previous year.

- a) Go to the worksheet “Sheet6”, rename it appropriately, make the usual heading in cells A1:A4, then click on the “GENERATE DATA” button to get your data.
- b) Can you conclude, at the 7% level of significance, that the annual mileage of Canadians is less than 20 000 km? Use the  $p$ -value approach.

7								
8	<b>Mileage (km)</b>		<i>Mileage (km)</i>		<b>Step 1</b>	Assumptions: Population normally Distributed		
9	24913				<b>Step 2</b>	$H_0: \mu =$	20000	km
10	22085	Mean	16860.1905			$H_A: \mu <$	20000	km
11	26473	Standard Error	1303.32352		<b>Step 3</b>	Test statistic: $t$ with	df =	20
12	17234	Median	17234			Right-tailed Test	$\alpha =$	7%
13	17748	Mode	#N/A		<b>Step 4</b>	$t =$	-2.41	
14	14372	Standard Deviation	5972.57869			$p$ -value =	0.0129	
15	14647	Sample Variance	35671696.2		<b>Step 5</b>	$p$ -value > $\alpha$		
16	15011	Kurtosis	-0.0359932			Fail to reject $H_0$		
17	29955	Skewness	0.38836149			There is not sufficient evidence, at the 5% level		
18	7360	Range	22595			of significance to conclude that the annual mileage		
19	17437	Minimum	7360			is less than 20 000 km.		
20	7451	Maximum	29955					
21	22409	Sum	354064					
22	13766	Count	21					
23	8571	Confidence Level(95.0%)	2718.68522					
24	10111							