

MATHEMATICS 201-510-LW

Business Statistics

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Fall 2008

Assignment 2 SOLUTIONS

This assignment is due on **Friday September 26** at the beginning of the class.

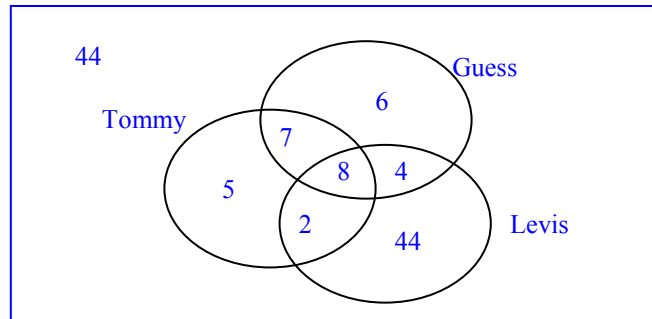
Question 1 (5 points)

In a marketing research, 120 people in a shopping center were interviewed to discover their buying preference concerning jeans. The following results were obtained.

- 22 buy Tommy Jeans;
- 25 buy Guess Jeans;
- 58 buy Levis;
- 15 buy Tommy and Guess Jeans;
- 10 buy Tommy Jeans and Levis;
- 12 buy Guess Jeans and Levis;
- 8 buy all three.

If a person from the 120 people interview is chosen at random, what is the probability that

- a) the person wears only Levis; $\frac{44}{120} = \frac{11}{30} \approx 0.3667$



- b) the person wears at least one of the above three labels $\frac{76}{120} = \frac{19}{30} \approx 0.6333$

Question 2 (4 points)

A soldier in Afghanistan has a 0.5% chance of hitting a mine during a patrol.

- a) What is the probability that he will hit a mine on his 4th patrol (thus he will not hit any on his first three patrols)?

$$\begin{aligned} P(N_1 N_2 N_3 H_4) &= P(N_1) P(N_2) P(N_3) P(H_4) \\ &= (1 - 0.005)^3 \cdot 0.005 \\ &= 0.004925 \end{aligned}$$

- b) What is the probability that he will hit no mines in his 20 patrols?

$$(1 - 0.005)^{20} = 0.995^{20} = 0.9046$$

Question 3 (10 points)

A poker hand is a subset of five cards drawn from a deck of 52 cards. Find the probability that the poker hand has

a) Only diamonds.

$$\frac{C_5^{13}}{C_5^{52}} = \frac{1287}{2598960} = \frac{33}{66640} \approx 0.0004952$$

b) Two queens and three aces.

$$\frac{C_2^4 C_3^4}{C_5^{52}} = \frac{24}{2598960} = \frac{1}{108290} \approx 0.000009234$$

c) Exactly four diamonds.

$$\frac{C_4^{13} C_1^{39}}{C_5^{52}} = \frac{715 \cdot 39}{2598960} = \frac{143}{13328} \approx 0.01073$$

d) Four queens.

$$\frac{C_4^4 C_1^{48}}{C_5^{52}} = \frac{48}{2598960} = \frac{1}{54145} \approx 0.00001847$$

e) Four of a kind.

$$\frac{C_1^{13} C_1^{48}}{C_5^{52}} = \frac{13 \cdot 48}{2598960} = \frac{1}{4165} \approx 0.0002401$$

Question 4 (7 points)

In a sales effectiveness seminar, a group of sales representatives tried two approaches to selling a customer a new automobile: the aggressive approach and the passive approach. The following record was kept.

	Sale	No Sale	
Aggressive	145	258	403
Passive	235	162	397
	380	420	800

Suppose that a customer is selected at random from all of the participant customers. Let us use the following notation for events: A = aggressive approach, Pa = passive approach, S = sale and N = no sale.

a) Find $P(S) = \frac{380}{800} = \frac{19}{40}$

b) Find $P(S, \text{ given } Pa) = \frac{235}{397}$

c) Are the events S and Pa independent? Explain.

No since $P(S) = \frac{19}{40} \neq P(S | Pa) = \frac{235}{397}$

d) Find $P(A \text{ and } S) = \frac{145}{800} = \frac{29}{160}$

e) Are the events A and S mutually exclusive? Explain.

No since $P(A \text{ and } S) = \frac{29}{160} \neq 0$

f) Find $P(A \text{ or } S) = P(A) + P(S) - P(A \text{ and } S) = \frac{403}{800} + \frac{380}{800} - \frac{145}{800} = \frac{638}{800} = \frac{319}{400}$.

Question 5 (8 points)

A car manufacturer runs two assembly lines, A and B . It has been observed that 90% of line A 's cars pass inspections, while 80% of line B 's cars pass inspection. In the past month, 320 cars were assembled on line A while 250 cars were assembled on line B . A car is selected at random.

- a) What is the probability it was assembled on line A ?

$$P(A) = \frac{320}{250+320} = \frac{320}{570} = \frac{32}{57}$$

- b) What is the probability a car passes inspection?

$$\begin{aligned} P(I) &= P(AI \text{ or } BI) = P(A)P(I|A) + P(B)P(I|B) \\ &= \frac{320}{570} \cdot 0.9 + \frac{250}{570} \cdot 0.8 \\ &= \frac{488}{570} = \frac{244}{285} \end{aligned}$$

- c) What is the probability a car passes inspection or comes from line A ?

$$\begin{aligned} P(I \text{ or } A) &= P(I) + P(A) - P(AI) \\ &= \frac{488}{570} + \frac{320}{570} - \frac{320}{570} \cdot 0.9 \\ &= \frac{520}{570} = \frac{52}{57} \end{aligned}$$

- d) What is the probability was assembled on line A if it passed inspection?

$$\begin{aligned} P(A|I) &= \frac{P(AI)}{P(I)} \\ &= \frac{\frac{320}{570} \cdot 0.9}{\frac{488}{570}} \\ &= \frac{288}{488} = \frac{36}{61} \end{aligned}$$

Question 6 (4 points)

In Quebec, 80% of the population have French as a mother tongue, 8% have English and 12% have another language. Also 25% of Quebecers with French as a mother tongue speak an other language, 95% of Quebecers with English as a mother tongue speak an other language and 85% of Quebecers who have neither French nor English as a mother tongue speak an other language. A Quebecer is selected at random.

- a) What is the probability that he speaks a second language?

$$\begin{aligned} P(S) &= P(F)P(S|F) + P(E)P(S|E) + P(O)P(S|O) \\ &= (0.8)(0.25) + (0.08)(0.95) + (0.12)(0.85) \\ &= 0.378 \end{aligned}$$

- b) What is the probability that his mother tongue is French given that he speaks a second language?

$$\begin{aligned} P(F|S) &= \frac{P(F \text{ and } S)}{P(S)} = \frac{P(F)P(S|F)}{P(S)} \\ &= \frac{0.8 \cdot 0.25}{0.378} \\ &= 0.529 \end{aligned}$$

Question 7 (3 points)

A car insurance company has established the following information from its accident records:

1. An insured drive has a probability of 0.07 of having a car accident.
2. If an accident has occurred, the probability the damage will amount to 25% of the car value is 0.75, that is will amount to 50% of the car value is 0.15, that it will amount to 75% of the car value is 0.08 and that it will amount to 100% of the car value is 0.02.

What base figure (before overhead and profit) should the company use in establishing the annual premium to charge for insuring a \$30 000 car?

$$\begin{aligned}
 E[x] &= \sum xp(x) \\
 &= 0 \cdot 0.93 + (0.25 \cdot 30000) \cdot (0.07 \cdot 0.75) + (0.50 \cdot 30000) \cdot (0.07 \cdot 0.15) \\
 &\quad + (0.75 \cdot 30000) \cdot (0.07 \cdot 0.08) + (30000) \cdot (0.07 \cdot 0.02) \\
 &= 719.25
 \end{aligned}$$

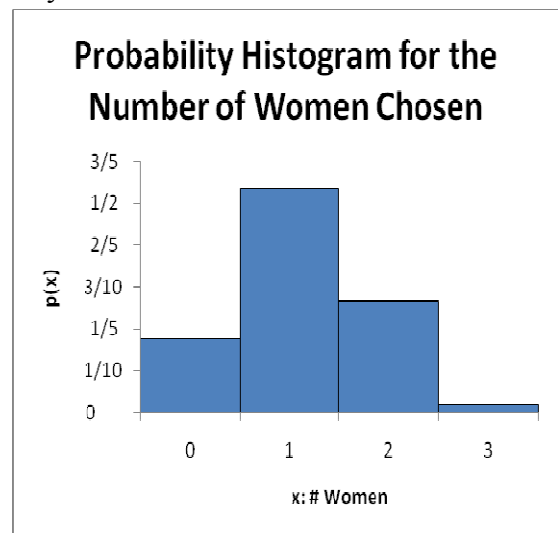
Hence the base amount should be \$719.25

Question 8 (10 points)

A company has 8 applicants for a job, five men and three women. Suppose that the eight applicants are equally qualified and that no preference is given for choosing either gender, and that there are three positions to fill. Let x be the number of women chosen to fill the three positions.

- a) Find the probability distribution for x .
- b) Draw a histogram of the probability distribution.
- c) Find the mean of this probability distribution.
- d) Find the standard deviation for this probability distribution.

Probability Distribution for the Number of Women Chosen	
x : # Women	$p(x)$
0	$\frac{C_3^5}{C_3^8} = \frac{5}{28}$
1	$\frac{C_1^3 C_2^5}{C_3^8} = \frac{15}{28}$
2	$\frac{C_2^3 C_1^5}{C_3^8} = \frac{15}{56}$
3	$\frac{C_3^3}{C_3^8} = \frac{1}{56}$



$$\mu = \sum xp(x) = 0 \cdot \frac{5}{28} + 1 \cdot \frac{15}{28} + 2 \cdot \frac{15}{56} + 3 \cdot \frac{1}{56} = \frac{9}{8} \text{ women}$$

$$\sigma^2 = \sum x^2 p(x) - \mu^2 = \left(0^2 \cdot \frac{5}{28} + 1^2 \cdot \frac{15}{28} + 2^2 \cdot \frac{15}{56} + 3^2 \cdot \frac{1}{56}\right) - \left(\frac{9}{8}\right)^2 = \frac{99}{56} - \frac{81}{64} = \frac{225}{448} \text{ women}^2$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{\frac{1}{2}} \approx 0.7087 \text{ women}$$

Question 9 (5 points)

An insurance company has found that the probability that a life insurance applicant will qualify at a special rates is 0.2. For the next 8 applicants for life insurance, what is the probability that

- a) Exactly three applicants will qualify at a special rate.

$$B(8, 0.20)$$

$$P(3) = C_3^8 (0.20)^3 (0.80)^5 = 0.1468$$

- b) At least one applicant will qualify at a special rate.

$$P(r \geq 1) = 1 - P(0)$$

$$= 1 - C_0^8 (0.80)^0 (0.20)^8$$

$$= 0.8322$$

- c) How many of the applicants do you expect to qualify at a special rate?

$$\mu = np = 0.2 \cdot 8 = 1.6 \text{ applicants}$$

Question 10 (4 points)

The number of students arriving late in class follows a Poisson distribution with a mean of 2 late arrivals per course.

- a) Find the probability that 3 or 4 students will be late in one class.

$$\mu = 2$$

$$P(3 \text{ or } 4) = P(3) + P(4) = \frac{2^3 e^{-2}}{3!} + \frac{2^4 e^{-2}}{4!} = 0.2707$$

- b) Find the probability that 12 students in total will be late during one week (5 classes).

$$\mu = 2 \cdot 5 = 10$$

$$P(12) = \frac{10^{12} e^{-10}}{12!} = 0.09478$$