

**MATHEMATICS 201-203-RE**

Integral Calculus

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**XXI – Alternating Series, Absolute Convergence and the Ratio Test**

1. Determine whether the alternating series converges or diverges.

a)  $\sum_{k=1}^{\infty} \frac{(-1)^k}{2k+1}$

b)  $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2}$

c)  $\sum_{k=1}^{\infty} \frac{(-1)^k}{5^k}$

d)  $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k+1}{k+3}$

e)  $\sum_{k=1}^{\infty} \frac{(-1)^k k}{\sqrt{k^2+1}}$

f)  $\sum_{k=1}^{\infty} \frac{(-1)^k}{\ln(k+2)}$

2. Show that the series is convergent and approximate the sum correct to four decimal places.

a)  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4}$

b)  $\sum_{n=1}^{\infty} 5\left(\frac{-3}{8}\right)^n$

c)  $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{10^n}$

d)  $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2^n n!}$

3. Determine whether each series converges absolutely, conditionally or diverges.

a)  $\sum_{k=1}^{\infty} k^2 \left(\frac{2}{5}\right)^k$

b)  $\sum_{k=1}^{\infty} \frac{(-1)^k 5^k}{k 2^k}$

c)  $\sum_{k=1}^{\infty} \frac{(-4)^k}{k^2}$

d)  $\sum_{k=1}^{\infty} \frac{k!}{3^k}$

e)  $\sum_{k=1}^{\infty} \frac{k^2+1}{k!}$

f)  $\sum_{k=1}^{\infty} \frac{5^k}{(3k)!}$

g)  $\sum_{k=1}^{\infty} \frac{(-1)^k k}{k^2+5}$

h)  $\sum_{k=1}^{\infty} \frac{(k+2)!}{k^3 k!}$

i)  $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\ln(k+2)}$

j)  $\sum_{k=1}^{\infty} \left(\frac{k}{3^k}\right)^2$

k)  $\sum_{k=1}^{\infty} \frac{2 \cdot 4 \cdot 6 \cdots (2k)}{1 \cdot 3 \cdot 5 \cdots (3k-2)}$

l)  $\sum_{k=1}^{\infty} \frac{(-1)^k k!}{1 \cdot 3 \cdot 5 \cdots (2k-1)}$

m)  $\sum_{k=1}^{\infty} \frac{(-1)^k}{(k+1)\ln(k+1)}$

n)  $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k+1} + \sqrt{k}}$

o)  $\sum_{k=1}^{\infty} \frac{(-1)^{k+1} 3^{2k-1}}{k^2+1}$

p)  $\sum_{k=1}^{\infty} (-1)^k k e^{-k}$

q)  $\sum_{k=1}^{\infty} \frac{(-1)^k (k+2)}{k^2+k}$

r)  $\sum_{k=1}^{\infty} \frac{(-1)^k \cos(k\pi)}{k}$

s)  $\sum_{k=1}^{\infty} \frac{(-2)^k}{\sqrt{k}}$

t)  $\sum_{k=1}^{\infty} \frac{(-2)^{2k}}{k+2}$

u)  $\sum_{k=1}^{\infty} \frac{(-1)^k \ln k}{\sqrt{k}}$

v)  $\sum_{n=1}^{\infty} \left(\frac{2n-1}{n+5}\right)^n$

w)  $\sum_{n=1}^{\infty} \frac{(-4)^n}{3 \cdot 8 \cdot 13 \cdots (5n-2)}$

x)  $\sum_{n=1}^{\infty} \left(\frac{2n-1}{n+5}\right)^n$

y)  $1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \dots$

z)  $\frac{1}{2} + \frac{1 \cdot 2}{2 \cdot 5} + \frac{1 \cdot 2 \cdot 3}{2 \cdot 5 \cdot 8} + \frac{1 \cdot 2 \cdot 3 \cdot 4}{2 \cdot 5 \cdot 8 \cdot 11} + \dots$

4. Use any appropriate method to decide whether each series converges or diverges.

- a)  $\sum_{k=1}^{\infty} \arctan k$       b)  $\sum_{k=1}^{\infty} \frac{2^k}{4^k - 3}$       c)  $\sum_{k=1}^{\infty} \frac{2^k k^2}{k!}$
- d)  $\sum_{k=1}^{\infty} \frac{k+1}{k^3 - 3k}$       e)  $\sum_{k=1}^{\infty} \frac{\ln k}{k^2}$       f)  $\sum_{k=1}^{\infty} \frac{2^k}{k!}$
- g)  $\sum_{k=1}^{\infty} \frac{k(-3)^k}{5^{k+1}}$       h)  $\sum_{k=1}^{\infty} \frac{(-1)^k \sqrt{k}}{k+1}$       i)  $\sum_{k=1}^{\infty} \frac{4^k}{k^2 + 1}$
- j)  $\sum_{k=1}^{\infty} \frac{1}{k^2}$       k)  $\sum_{k=1}^{\infty} \frac{1}{5^k}$       l)  $\sum_{k=1}^{\infty} \frac{3^{2k}}{5^{k-2}}$
- m)  $\frac{1}{2} - \frac{4}{9} + \frac{9}{28} - \frac{16}{65} + \frac{25}{126} - \frac{36}{217} + \dots$       n)  $\frac{1}{2} + \frac{1.7}{2.4} + \frac{1.7 \cdot 13}{2.4 \cdot 6} + \frac{1.7 \cdot 13 \cdot 19}{2.4 \cdot 6 \cdot 8} + \dots$
- o)  $\sum_{n=2}^{\infty} \frac{5^n}{(\ln n)^n}$

## Answers

1. a) Converges      b) Converges      c) Converges  
 d) Diverges      e) Diverges      f) Converges
2. a) 0.9470      b) -1.3636      c) -0.0676      d) 0.3935
3. a) Converges absolutely      b) Diverges      c) Diverges  
 d) Diverges      e) Converges absolutely      f) Converges absolutely  
 g) Converges conditionally      h) Diverges      i) Converges conditionally  
 j) Converges absolutely      k) Converges absolutely      l) Converges absolutely  
 m) Converges conditionally      n) Converges conditionally      o) Diverges  
 p) Converges absolutely      q) Converges conditionally      r) Diverges  
 s) Diverges      t) Diverges      u) Converges Conditionally  
 v) Diverges      w) Converges absolutely      x) Converges absolutely  
 y) Converges absolutely      z) Converges absolutely
4. a) Diverges      b) Converges      c) Converges      d) Converges  
 e) Converges      f) Converges      g) Converges      h) Converges  
 i) Diverges      j) Converges      k) Diverges      l) Diverges  
 m) Converges      n) Diverges      o) Converges