

**MATHEMATICS 201-203-RE**

Integral Calculus

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**XX – The Integral and Comparison Tests**

1. Use the integral test to determine whether the series is convergent or divergent.

a)  $\sum_{k=1}^{\infty} \frac{1}{k^4}$

b)  $\sum_{k=1}^{\infty} \frac{2}{k\sqrt[3]{k}}$

c)  $\sum_{k=1}^{\infty} \frac{1}{k^2+9}$

d)  $\sum_{k=1}^{\infty} \frac{5k}{3k^2+7}$

e)  $\sum_{n=1}^{\infty} \left(\frac{4}{n}\right)^2$

f)  $\sum_{k=1}^{\infty} e^{-k}$

g)  $\sum_{k=1}^{\infty} \frac{\ln k}{k}$

h)  $\sum_{k=1}^{\infty} \frac{1}{k \ln k}$

i)  $\sum_{k=1}^{\infty} \frac{\arctan k}{k^2+1}$

j)  $\sum_{k=1}^{\infty} \frac{1}{(2k+1)(3k+1)}$

k)  $\sum_{k=1}^{\infty} \frac{k}{2^k}$

l)  $\sum_{k=1}^{\infty} ke^{-k^2}$

m)  $1 + \frac{1}{8} + \frac{1}{27} + \frac{1}{64} + \frac{1}{125} + \dots$

n)  $1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \frac{1}{5\sqrt{5}} + \dots$

o)  $\frac{1}{\sqrt{1+1}} + \frac{1}{2\sqrt{4+1}} + \frac{1}{3\sqrt{9+1}} + \frac{1}{4\sqrt{16+1}} + \dots$

p)  $\sum_{n=3}^{\infty} \frac{1}{n \ln n \ln(\ln n)}$

2. For what values of  $p$  do the following series converge?

a)  $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^p}$

b)  $\sum_{n=1}^{\infty} n(1+n^2)^p$

3. Determine whether the series is convergent or divergent.

a)  $\sum_{k=1}^{\infty} \frac{k^2}{k^4+2k+2}$

b)  $\sum_{k=1}^{\infty} \frac{k^3-4}{5k^4+2k-1}$

c)  $\sum_{k=1}^{\infty} \frac{2}{1+5^k}$

d)  $\sum_{n=1}^{\infty} \frac{5}{n2^n}$

e)  $\sum_{k=1}^{\infty} \frac{k}{\sqrt{k^6+1}}$

f)  $\sum_{k=1}^{\infty} \frac{1}{\sqrt[3]{k^2+3k}}$

g)  $\sum_{k=1}^{\infty} \frac{\sqrt{k}}{k+2}$

h)  $\sum_{k=1}^{\infty} \frac{2^k}{4+7^k}$

i)  $\sum_{k=1}^{\infty} \frac{n+5}{\sqrt[3]{n+n^2+n^7}}$

j)  $\sum_{k=1}^{\infty} \frac{1+3^k}{2+4^k}$

k)  $\sum_{k=1}^{\infty} \frac{5+\cos k}{5^k}$

l)  $\sum_{k=1}^{\infty} \frac{\sin^2 k}{9^k}$

m)  $\sum_{k=1}^{\infty} \frac{n!}{n^n}$

n)  $\sum_{k=1}^{\infty} \frac{1}{n!}$

o)  $\sum_{k=1}^{\infty} \frac{3+2^k}{k+5^k}$

p)  $\sum_{n=2}^{\infty} \left( \frac{\sqrt{n}}{n^3-1} - \frac{\sqrt[4]{n}}{\sqrt{n+1}} \right)$

q)  $\sum_{n=1}^{\infty} \left( \frac{2}{5+4^n} - \frac{n^2}{4n^5-n+1} \right)$

r)  $\sum_{n=1}^{\infty} \left( \frac{n^2}{n^3-4} - \frac{n^3}{n^4+3} \right)$

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## Answers

1. a) Converges  
d) Diverges  
g) Diverges  
j) Converges  
m) Converges  
p) Diverges
  2. a) If  $p > 1$
  3. a) Converges  
d) Converges  
g) Diverges  
j) Converges  
m) Converges  
p) Diverges
- |                |              |
|----------------|--------------|
| b) Converges   | c) Converges |
| e) Converges   | f) Converges |
| h) Diverges    | i) Converges |
| k) Converges   | l) Converges |
| n) Converges   | o) Converges |
| b) If $p < -1$ |              |
| b) Diverges    | c) Converges |
| e) Converges   | f) Diverges  |
| h) Converges   | i) Converges |
| k) Converges   | l) Converges |
| n) Converges   | o) Converges |
| q) Converges   | r) Converges |