MATHEMATICS 201-203-RE
Integral Calculus
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Winter 2009

XVIII – Sequences

1. List the first five terms of the sequence.

a) $a_n = n^2 + 1$

b) $a_n = (-1)^{n+1}(2n+1)$

c) $\left\langle \dfrac{(2n)!}{3^{2n-1}} \right\rangle_{n=1}^{\infty}$

d) $a_1 = 1, \quad a_{n+1} = \dfrac{1}{1+a_n}$

2. Find a formula for the general term of the sequence.

a) $1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots$

b) $1, -3, 9, -27, 81, \ldots$

c) $1, 0, 1, 0, 1, 0, \ldots$

d) $2, 7, 12, 17, 22, \ldots$

e) $1, \frac{5}{3}, \frac{7}{3}, 3, \frac{11}{3}, \ldots$

f) $10, 4, \frac{6}{5}, \frac{8}{25}, \frac{2}{25}, \ldots$

3. Determine whether the sequence converges or diverges. If it converges, find the limit.

a) $a_n = n^2 + 1$

b) $a_n = \dfrac{2n-1}{2n+1}$

c) $a_n = \dfrac{n}{3^n}$

d) $a_n = (-1)^n \dfrac{n}{1+\sqrt{n}}$

e) $b_n = \cos \frac{\pi n}{2}$

f) $\left\langle \dfrac{4^n}{7^{n+1}} \right\rangle_{n=1}^{\infty}$

g) $a_n = \dfrac{\ln\left(\frac{1}{n}\right)}{\ln(n+4)}$

h) $a_n = \dfrac{3n^2 - 4n + 1}{n^2 + 1}$

i) $a_n = \ln(n+1) - \ln n$

j) $a_n = \dfrac{2 + (-1)^n}{n^2}$

k) $a_n = \dfrac{1}{n^2} + \dfrac{2}{n^2} + \dfrac{3}{n^2} \ldots + \dfrac{n}{n^2}$

l) $a_n = \dfrac{1^2}{n^3} + \dfrac{2^2}{n^3} + \dfrac{3^2}{n^3} + \cdots + \dfrac{n^2}{n^3}$

m) $(\sqrt{2} - \sqrt{3}), (\sqrt{3} - \sqrt{4}), (\sqrt{4} - \sqrt{5}), \ldots$

n) $\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \ldots$

o) $a_n = \sqrt[n]{n}$

p) $a_n = \left(1 + \dfrac{1}{n}\right)^n$

4. Determine whether each sequence is increasing, decreasing or not monotonic and whether it is bounded. Indicate whether the sequence is convergent or divergent.

   a) $a_n = \left(\frac{2}{3}\right)^n$

   b) $a_n = \left(\frac{-2}{3}\right)^n$

   c) $a_n = \left(\frac{3}{2}\right)^n$

   d) $a_n = \frac{n+1}{n^2+1}$

   e) $a_n = 1 - 2^{-n}$

   f) $a_n = \frac{(-1)^n n}{n+1}$

5. A deposit of \$2000 is made in an account that earns 8%, compounded monthly. Find a sequence that represents the monthly balances. Use your result to find the amount of money in the account 5 years later.

6. A deposit of \$5000 is made in an account that earns 12%, compounded semi-annually. Find the sequence that represents the semi-annual balances. Use your result to find the amount of money in the account 5 years later.

7. If the average price of a new car increases by 3% per year and the average price is currently \$22 000, then find a sequence that represents the average price of a car n years from now.

8. Each year a machine loses 15% of the value it had at the beginning of the year. Find the value of the machine at the end of 10 years if it cost \$200 000 new.

9. A well-drilling company charges \$40 for drilling the first meter of a well, \$42 for the second meter, \$44 for the third meter, and so on. Determine the cost of drilling the n^{th} meter.

10. A person accepts a position with a company at a salary of \$42 000 for the first year. The person is guaranteed a raise of 5% per year. Find a sequence that represents the person's salary in the n^{th} year of employment. Use this result to determine his salary during the fifth year of employment.

Answers

1. a) 2, 5, 10, 17, 26
   b) 3, -5, 7, -9, 11
   c) $\frac{2}{3}, \frac{24}{27}, \frac{720}{243}, \frac{40320}{2187}, \frac{3628800}{19683}$
   d) $\frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{5}{8}, \frac{8}{13}$
2. a) $a_n = \frac{1}{2n-1}$
   b) $a_n = (-3)^{n-1}$
   c) $a_n = \frac{1+(-1)^{n+1}}{2}$
   d) $a_n = 5n-3$
   e) $a_n = \frac{2n+1}{3}$
   f) $a_n = \frac{2n}{5^{n-2}}$
3. a) Diverges
   b) Converges to 1
   c) Converges to 0
   d) Diverges
   e) Diverges
   f) Converges to 0
   g) Converges to -1
   h) Converges to 3
   i) Converges to 0
   j) Converges to 0
   k) Converges to ½
   l) Converges to $\frac{1}{3}$
   m) Converges to 0
   n) Converges to 2
   o) Converges to 1
   p) Converges to *e*
4. a) Decreasing, bounded by 0 and $\frac{2}{3}$ converges to 0.
   b) Nonmonotonic, bounded by $\frac{-2}{3}$ and $\frac{4}{9}$, converges to 0.
   c) Increasing, lower bound of $\frac{3}{2}$ but no upper bound (thus unbounded), diverges.
   d) Decreasing, bounded by 0 and 1, converges to 0.
   e) Increasing, bounded by ½ and 1, converges to 1.
   f) Nonmonotonic, bounded by –1 and 1, diverges.
5. $S = a_n = 2000\left(1+\frac{0.08}{12}\right)^n$ (*n* in months) $2979.69
6. $S = a_n = 5000(1.06)^n$ (*n* is in half-years) $8954.24
7. $P_n = 22000(1.03)^n$ (*n* is in years)
8. $V_n = 200\,000(0.85)^n$ (*n* is in years) $39 374.88
9. $C_n = 40 + 2(n-1)$ (*n* is in meters)
10. $S_n = 42000(1.05)^{n-1}$ (*n* is in years) $51051.26