

## MATHEMATICS 201-203-RE

Integral Calculus

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### XVI – Business Applications

1. The demand function for a certain DVD is  $p = 81 - x^2$ , where  $p$  is the unit price (in \$) and  $x$  the number of units (in thousands), whereas the supply function is  $p = x^2 + 4x + 11$ . Determine the consumers' surplus and the producers' surplus when the market unit price is set at the equilibrium price.

2. The demand function for a particular brand of coffee is  $p = \frac{12}{x+1}$ , where  $p$  is the price (in \$) per kilogram and  $x$  the number of kilograms (in thousands), and the supply function is  $p = 1 + 0.2x$ . Determine the consumers' surplus and the producers' surplus when the market unit price is set at the equilibrium price.

3. A manufacturer has determined that when  $x$  units of a particular commodity are produced, the price at which all the units can be sold is  $p = D(x)$  dollars per unit, where  $D$  is the demand function given by

$$D(x) = \frac{400}{0.5x + 2}$$

- How many units can the manufacturer expect to sell if the commodity is priced at \$16 per unit?
  - Find the consumers' surplus that corresponds to the level of production found in part (a).
4. The demand function for a certain brand of compact disc is given by  $p = -0.01x^2 - 0.2x + 11$  dollars per unit and  $q$  is the quantity demand each week, measured in units of a million, and the supply function is given by  $p = 0.01x^2 + 0.1x + 2$ .
- Determine the consumer's surplus if the wholesale market price is set at \$3/dics.
  - Determine the producer's surplus if the wholesale market price is set at \$3/dics.
  - Determine the producer's surplus if the wholesale market price is set at the equilibrium price.

5. The demand function for a certain USB key is  $p = \sqrt{490 - 6x}$ , where  $p$  is the price (in \$) per unit and  $x$  the number of units (in thousands), and the supply function is  $p = \sqrt{130 + 18x}$ . Determine the consumers' surplus and the producers' surplus when the market unit price is set at the equilibrium price.

6. Dwight is to receive a bursary for university of 12000 per year for the next three years. Assume a continuous income stream of income.
- Find the total income over the next three years.
  - Assuming the inflation rate is 2% compounded continuously, what is the present value of the income?
  - Assuming the inflation rate is 2% compounded continuously, what is the future value of the income?
7. A small brewery considers the output of its bottling machine as a continuous income stream with an annual rate of flow at time  $t$  given by
- $$f(t) = 80e^{-0.1t}$$
- in thousands of dollars.
- Find the income from this stream for the next 10 years.
  - Find the average income from this stream for the next 10 years.
  - Assuming the current interest rate is 8% compounded continuously, what is the present value of the income stream?
  - Assuming the current interest rate is 8% compounded continuously, what is the future value of the income stream?
8. In planning for retirement, you decide to invest \$200 a month in an account paying 6% per year compounded continuously. Assuming a continuous stream of investment and continuous compounding, how much will you have accumulated when you retire 35 years from now?
9. In planning for retirement, you decide to invest \$200 a month in an account paying 6% per year compounded continuously. You increase the amount you invest at a rate of 2% per year. Assuming a continuous stream of investment and continuous compounding, how much will you have accumulated when you retire 35 years from now? What is the present value?
10. Suppose that a printing firm considers the production of its presses as a continuous income stream. If the annual rate of flow at time  $t$  is given by  $f(t) = 97.5e^{-0.2t-0.6}$  in thousands of dollars. Find the present value and future value of the machines over the next 5 years if money is worth 10%, compounded continuously.
11. The management of Toronto Pizza is selling a 10-year franchise in Rimouski. Past experience in similar localities suggests that  $t$  years from now the franchise will be generating profit at the rate of  $f(t) = 10000 + 500t$  dollars per year. If the prevailing annual interest rate remains fixed at 10% compounded continuously, what is the present value of the franchise?

12. An oil well is being offered for sale for \$1 900 000 and has a rate of change of income per year given by  $f(t) = 100t^2 e^{-0.1t}$ , in thousands of dollars, and a useful life of five years. Would you buy if the current interest rate  $r$  that can be compounded continuously over the next five years is
- 10%?
  - 12%?
13. Suppose that a business provides a continuous income stream with an annual rate of flow at time  $t$  given by  $f(t) = 450e^{-0.09t}$  in thousands of dollars. If the annual interest rate is 6%, compounded continuously, find the capital value of the machinery.
14. It is estimated that  $t$  years from now an apartment complex will be generating profit for its owner at the rate of  $f(t) = 10000 + 500t$  dollars per year. If the profit is generated in perpetuity and the prevailing annual interest rate remains fixed at 10% compounded continuously, what is the present value of the apartment complex?
15. An investment with a continuous income flow of \$10 000 per year forever is being sold. Assume that the current annual interest rate of 10% compounded continuously will continue forever. Would you buy if the price was
- \$110 000?
  - \$90 000?
16. A certain country's income distribution is described by the function  $f(x) = \frac{15}{16}x^2 + \frac{1}{16}x$ . Find the Gini coefficient of income.
17. In a study conducted by a certain country's Economics Development Board, it was found that the Lorenz curve for the distribution of income of lawyers was described by the function  $f(x) = \frac{9}{11}x^4 + \frac{2}{11}x$  and that of college teachers by the function  $g(x) = \frac{13}{14}x^2 + \frac{1}{14}x$ . Compute the Gini coefficient of income for both professions. Which profession has a more equitable income distribution?
18. In an effort to make the distribution of income more equal, the government of a country passes a tax law that changes the Lorenz curve for one year from  $f(x) = x^{2.6}$  to  $f(x) = 0.57x^2 + 0.43x$  for the next year. Find the Gini coefficient of income for both years and determine whether the distribution of income is more or less equal after the tax law is passed. Interpret the result.

## Answers

1. Equilibrium point: (5,56)      CS = \$83 333.33      PS = \$133 333.33
2. Equilibrium point: (5,2)      CS = \$11 501.11      PS = \$2 500.00
3. a) 46      b) CS = \$1284.58
4. a) \$93 333 333.33      b) \$ 3 483 616.57      c) CS = \$45 000 000      PS = \$33 750 000
5. Equilibrium point: (15,20)      CS = \$ 16 290.26      PS = \$ 58 601.04
6. a) \$36 000      b) \$34 941.28      c) \$37 101.93
7. a) \$294 303.55      b) \$29 430.36      c) \$370 978.27      d) \$ 825 627.33
8. \$286 646.80
9. FV = \$368 160.20      PV = \$45 083.58
10. PV = \$138 565.44      FV = \$228 455.79
11. \$76 424.11
12. a) PV = \$2 007 535 Yes      b) PV = \$1 870 468 No
13. \$3 000 000
14. \$150 000
15. PV = \$100 000      (a) No      (b) Yes
16. Gini = 0.3125
17.  $Gini_{\text{Lawyers}} = 0.4909$        $Gini_{\text{College teachers}} = 0.3095$       College teachers
18.  $Gini_{\text{Before}} = 0.4444$        $Gini_{\text{after}} = 0.1900$   
The change decreases the difference in income.