MATHEMATICS 201-203-RE
Integral Calculus
Martin Huard
Winter 2009

XIX – Series

1. Write the fist four terms of each series, and then write out the first five terms of the sequence s_n of partial sums. Find a "simple" formula for the nth partial sum s_n in terms of n, determine whether the series converges or diverges, and if it converges, find its sum S.

   a) $\sum_{k=1}^{\infty} \frac{2^{k-1}}{4}$ b) $\sum_{k=0}^{\infty} \frac{2}{5^k}$

   c) $\sum_{k=1}^{\infty} \left(\frac{1}{k+4} - \frac{1}{k+5} \right)$ d) $\sum_{k=1}^{\infty} \frac{2}{k(k+2)}$

2. Determine whether the series is convergent or divergent. If it is convergent, find its sum.

   a) $\sum_{k=1}^{\infty} \left(-\frac{2}{3}\right)^k$ b) $\sum_{k=1}^{\infty} \frac{3 \cdot 4^{k-1}}{5^{k+1}}$

   c) $\sum_{k=1}^{\infty} \frac{4 \cdot 2^{2k-1}}{25^k}$ d) $\sum_{k=1}^{\infty} \frac{2 \cdot (-3)^{3k}}{5 \cdot (4)^{2k+3}}$

   e) $\sum_{k=1}^{\infty} \left(\frac{1}{2k-1} - \frac{1}{2k+1} \right)$ f) $\sum_{k=1}^{\infty} \frac{1}{k^2+5k+6}$

   g) $\sum_{k=1}^{\infty} \frac{1}{k^2+4k+3}$ h) $\sum_{k=2}^{\infty} \frac{1}{k^2-1}$

   i) $\sum_{k=1}^{\infty} \ln\left(\frac{k+1}{k+2} \right)$ j) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n+1}$

   k) $\sum_{k=1}^{\infty} \left(\frac{1}{2^{k+1}} + \frac{2}{3^{k-1}} \right)$ l) $\sum_{k=1}^{\infty} \left(1 - \frac{1}{k^2} \right)$

   m) $\sum_{k=1}^{\infty} \frac{(k+1)^3}{k^2+2k+3}$ n) $\sum_{k=0}^{\infty} \frac{5^{k+1}}{6^k}$

   o) $1 - \frac{5}{8} + \frac{25}{64} - \frac{125}{512} + \frac{625}{4096} - \ldots$ p) $\ln\left(1-\frac{1}{4}\right) + \ln\left(1-\frac{1}{9}\right) + \ln\left(1-\frac{1}{16}\right) + \ldots$

   q) $9 + 3 + 1 + \frac{1}{3} + \frac{1}{9} + \ldots$ r) $\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \frac{1}{7\cdot 9} + \ldots$

3. Express the number as a ratio of integers using series.

   a) $0.\overline{5}$ b) $3.\overline{215}$

   c) $0.654\overline{321}$ d) $0.451141414\ldots$

4. What is the value of c if $\sum_{n=0}^{\infty} (1+c)^{-n} = 3$?

5. Suppose that an employer offered to pay you 1 cent the first day, and double your wages each day thereafter. Find your total wages for working n days and for 20 days.

6. The annual spending by tourists in a resort is 100 million dollars. Approximately 75% of that revenue is again spent in the resort city, and of that amount approximately 75% is again spent in the resort city. If this pattern continues, write the geometric series that gives the total amount of spending generated by the 100 million dollars and find the sum of this series.

7. A manufacturer sells 50 000 units of a product each year. In any given year, each unit has a 20% chance of breaking. How many units will be in use in n years? Find the market stabilization level of the product.

8. A deposit of $100 is made at the end of each month for 5 years into an account that pays 10% interest, compounded monthly. Find the balance in the account at the end of 5 years.

9. A deposit of $200 is made at the end of each month in an account that earns 6% interest, compounded monthly. Find the balance in the account after n months and after 10 years.

10. In planning for retirement, you decide to invest $200 a month in an account paying 6% interest per year compounded monthly. How much will you have accumulated when you retire 35 years from now? What is the present value?

11. If you wish to have $1 000 000 when you retire in 40 years, how much money should you place each month in an account, if the interest rate is 4% compounded monthly?

12. In planning for retirement, you decide to invest $300 a month in an account paying 8% interest per year compounded monthly. How much will you have accumulated when you retire 25 years from now? Suppose that you do not want to save money right away, but wait 10 years. What will then be the monthly payments if you are to have the same amount of money when you retire (25 years from now)?

13. Suppose that Paul just won a lottery. He is offered either a lump sum of 5 million dollars now, or $40 000 a month for life. If Paul is 35 years old (and his life expectancy is 40 more years), which option should he choose, assuming money is worth 6% compounded monthly. What if Paul is 60 years old (and his life expectancy is 15 more years)?

14. Mary just bought a condo for $150 000. It the annual interest rate is 5% compounded monthly, what should be her monthly payments if she takes a 25 year mortgage on the $150000?

15. Greg just bought a house for $350 000. It the annual interest rate is 4% compounded monthly, what will be his monthly payments if he takes a 30 year mortgage and gives a cash down of $20 000?

Answers

1. a) $\sum_{k=1}^{\infty} \frac{2^{k-1}}{4} = \frac{1}{4} + \frac{2}{4} + \frac{4}{4} + \frac{8}{4} + \dots$ $\langle s_n \rangle = \frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{15}{4}, \frac{31}{4}, \dots$

   $s_n = \frac{2^n - 1}{4}$ Diverges

   b) $\sum_{k=0}^{\infty} \frac{2}{5^k} = 2 + \frac{2}{5} + \frac{2}{25} + \frac{2}{125} + \dots$ $\langle s_n \rangle = 2, \frac{12}{5}, \frac{62}{25}, \frac{312}{125}, \frac{1562}{625}, \frac{7812}{3125}, \dots$

   $s_n = \frac{\frac{5^{n+1}-1}{2}}{5^n} = \frac{5}{2} - \frac{5}{2}\left(\frac{1}{5}\right)^n$ $S = \frac{5}{2}$

   c) $\sum_{k=1}^{\infty} \left(\frac{1}{k+4} - \frac{1}{k+5} \right) = \left(\frac{1}{5} - \frac{1}{6}\right) + \left(\frac{1}{6} - \frac{1}{7}\right) + \left(\frac{1}{7} - \frac{1}{8}\right) + \left(\frac{1}{8} - \frac{1}{9}\right) + \dots$

   $\langle s_n \rangle = \frac{1}{5} - \frac{1}{6}, \frac{1}{5} - \frac{1}{7}, \frac{1}{5} - \frac{1}{8}, \frac{1}{5} - \frac{1}{9}, \frac{1}{5} - \frac{1}{10}, \dots$ $s_n = \frac{1}{5} - \frac{1}{n+5}$ $S = \frac{1}{5}$

   d) $\sum_{k=1}^{\infty} \frac{1}{k(k+2)} = \sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{k+2} \right) = \left(1 - \frac{1}{3}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \left(\frac{1}{4} - \frac{1}{6}\right) + \dots$

   $\langle s_n \rangle = 1 - \frac{1}{3}, 1 - \frac{1}{3} + \frac{1}{2} - \frac{1}{4}, 1 - \frac{1}{4} + \frac{1}{2} - \frac{1}{5}, 1 - \frac{1}{5} + \frac{1}{2} - \frac{1}{6}, 1 - \frac{1}{6} + \frac{1}{2} - \frac{1}{7}, \dots$

   $s_n = 1 - \frac{1}{n+1} + \frac{1}{2} - \frac{1}{n+2}$ $n > 1$ $S = \frac{3}{2}$

2. a) $\frac{-2}{5}$ b) $\frac{3}{5}$ c) $\frac{8}{21}$ d) Diverges e) 1 f) $\frac{1}{3}$
   g) $\frac{5}{12}$ h) $\frac{3}{4}$ i) Diverges j) Diverges k) $\frac{7}{2}$ l) Diverges
   m) Diverges n) 30 o) $\frac{8}{13}$ p) $-\ln 2$ q) $\frac{27}{2}$ r) $\frac{1}{2}$

3. a) $\sum_{k=1}^{\infty} \frac{5}{10^k} = \frac{5}{9}$ b) $3 + \sum_{k=1}^{\infty} \frac{215}{1000^k} = \frac{3212}{999}$

   c) $\frac{654}{1000} + \sum_{k=1}^{\infty} \frac{321}{1000^{k+1}} = \frac{217889}{333000}$ d) $\frac{451}{1000} + \sum_{k=1}^{\infty} \frac{1}{1000} \frac{14}{100^k} = \frac{44663}{99000}$

4. $c = \frac{1}{2}$

5. $\sum_{k=1}^{n} (0.01) 2^{k-1} = \frac{2^n - 1}{100}$ Total wages : \$ 10 485.75

6. $\sum_{k=1}^{\infty} 100(0.75)^{k-1} = 400$ million dollars

7. $\sum_{k=0}^{n} 50\,000(0.80)^k = 250\,000\left(1 - 0.80^{n+1}\right)$ Market stabilization level is 250 000 units.

8. \$7743.71

9. $S_n = 40\,000\left(1.005^n - 1\right)$ \$32 775.87

10. S = \$284 942.06 A = \$35 076.05

11. R = \$856.05

12. S = \$285 307.95 R = \$824.50

13. Since the present value for the payments is A = \$7 269 903.37, he should take the payments. If Paul is 60, then A = \$4 740 140.59, so he should take the 5 million.

14. \$876.89 per month

15. \$1575.47 per month