

MATHEMATICS 201-203-RE

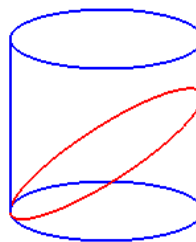
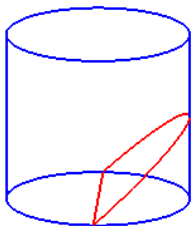
Integral Calculus

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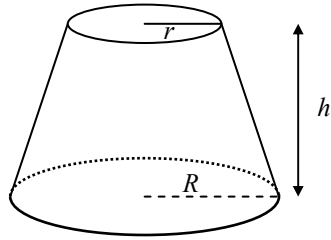
XIV – Volumes by the Slicing Method

1. A certain solid has a circular base of radius 3 units. If cross sections perpendicular to one of the diameters of the base are squares, find the volume of the solid.
2. A monument is 30 meters high. A horizontal cross section x meters above the base is an equilateral triangle whose sides are $\frac{1}{15}(30-x)$ meters long. Find the volume of the monument.
3. The Department of Public Works intends to cut down a diseased elm tree one meter in diameter. They first cut out a wedge bounded below by a horizontal plane and bounded above by a plane that meets the horizontal plane at a 45° angle. What is the volume of the wedge if the planes meet
 - a) along a diameter of the tree?
 - b) on the side of the tree?

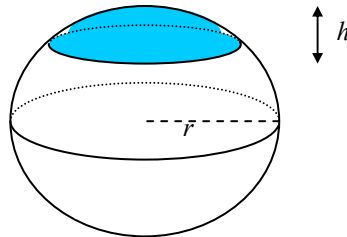


4. Find the volume of the solid whose base is the region bounded by the curves $y = x$ and $y = x^2$, and whose cross-sections perpendicular to the x -axis are squares.
5. Find the volume of the solid whose base is enclosed by the circle $x^2 + y^2 = 1$ and whose cross sections taken perpendicular to the base are
 - a) semicircles
 - b) squares
 - c) equilateral triangles
 - d) isosceles right triangles (with hypotenuse on base).
6. Find the volume of the solid whose base is bounded by the graphs of $y = x+1$ and $y = x^2 - 1$ and whose cross sections taken perpendicular to the x -axis are
 - a) squares
 - b) rectangles of height 1.
7. Find the volume of a right circular cone with height h and base radius r .

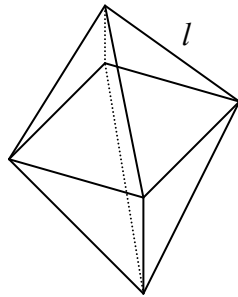
8. A dome is designed with three semicircular supports of radius r so that each horizontal cross section is a regular hexagon. Find the volume of the dome.
9. Find the volume common to two spheres, each with radius r , if the center of each sphere lies on the surface of the other sphere.
10. Find the volume of a frustum of a right circular cone with height h , lower base radius R and top radius r .



11. Find the volume for the cap of a sphere with radius r and height h .



12. Find the volume of a frustum of a pyramid with square base of side b , square top of side a , and height h .
13. A regular **octahedron** is a solid bounded by eight congruent equilateral triangles. Find the volume of an octahedron if each of its eight bounding triangles has sides of length l .



14. Find the volume of a pyramid with height h and base an equilateral triangle with side a (a tetrahedron).

Answers

- 144
- $10\sqrt{3} \text{ m}^3$
- a) $\frac{1}{12} \text{ m}^3$ b) $\frac{\pi}{8} \text{ m}^3$
- $\frac{1}{30}$
- a) $\frac{2\pi}{3}$ b) $\frac{16}{3}$ c) $\frac{4\sqrt{3}}{3}$ d) $\frac{4}{3}$
- a) $\frac{81}{10}$ b) $\frac{9}{2}$
- $\frac{1}{3}\pi r^2 h$
- $\sqrt{3}r^3$
- $\frac{5}{12}\pi r^3$
- $\frac{1}{3}\pi h(R^2 + Rr + r^2)$
- $\pi h^2\left(r - \frac{h}{3}\right)$
- $\frac{1}{3}h(a^2 + ab + b^2)$
- $\frac{\sqrt{2}}{3}l^3$
- $\frac{\sqrt{3}}{12}a^2h$