

MATHEMATICS 201-203-RE

Integral Calculus

Martin Huard

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XI – Improper Integrals

1. Evaluate each improper integral (if it is convergent).

a) $\int_1^{\infty} \frac{1}{x\sqrt{x}} dx$

b) $\int_2^{\infty} \frac{1}{4+x^2} dx$

c) $\int_2^{\infty} \frac{2}{x^2-1} dx$

d) $\int_0^{\infty} \frac{1}{(x+2)(x+3)} dx$

e) $\int_0^{\infty} e^{-x} \sin x dx$

f) $\int_2^{\infty} \frac{1}{x\sqrt{\ln x}} dx$

g) $\int_{-\infty}^{-2} \frac{1}{(x-1)^4} dx$

h) $\int_{-\infty}^{\infty} x^3 dx$

i) $\int_{-\infty}^{\infty} \frac{x}{(x^2+3)^2} dx$

j) $\int_0^{\infty} \cos^2 \theta d\theta$

k) $\int_{-\infty}^{\infty} \frac{e^{-t}}{1+e^{-2t}} dt$

l) $\int_0^{\infty} x^2 e^{-x} dx$

2. Evaluate each improper integral (if it is convergent).

a) $\int_0^9 \frac{1}{x\sqrt{x}} dx$

b) $\int_{\frac{1}{2}}^5 \frac{1}{(2x-1)^2} dx$

c) $\int_{-1}^1 \frac{1}{x^3} dx$

d) $\int_0^{\frac{\pi}{2}} \tan x dx$

e) $\int_0^1 \frac{x}{\sqrt{1-x^2}} dx$

f) $\int_0^4 \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx$

g) $\int_{-1}^8 x^{-\frac{1}{3}} dx$

h) $\int_1^3 \frac{x}{2-x} dx$

i) $\int_1^4 \frac{1}{x^2-4} dx$

j) $\int_0^{\frac{\pi}{4}} \frac{\sec^2 t}{\sqrt{1-\tan t}} dt$

k) $\int_{-\infty}^{\infty} \frac{1}{x^5} dx$

l) $\int_{-\ln 2}^{\ln 2} \frac{e^x}{\sqrt[5]{e^x-1}} dx$

m) $\int_{-\infty}^{\infty} \frac{1}{(x-1)^2} dx$

n) $\int_0^{\infty} \frac{1}{x^2-4} dx$

o) $\int_0^{\infty} \frac{1}{\sqrt{x}(1+x)} dx$

3. Find the value of the constant A for which the given integral converges, then evaluate the integral for this value of A .

$$\int_0^{\infty} \left(\frac{4}{x+5} - \frac{A}{x+3} \right) dx$$

Answers

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|-----------------------------|------------------------|--------------------|--|
| 1. a) 2 | b) $\frac{\pi}{8}$ | c) $\ln 3$ | d) $\ln 3 - \ln 2$ |
| e) 1 | f) Diverges | g) $\frac{1}{81}$ | h) Diverges |
| i) 0 | j) Diverges | k) $\frac{\pi}{2}$ | l) 2 |
| 2. a) Diverges | b) Diverges | c) Diverges | d) Diverges |
| e) $\frac{\pi}{2}$ | f) $2 - \frac{2}{e^2}$ | g) $\frac{9}{2}$ | h) Diverges |
| i) Diverges | j) 2 | k) Diverges | l) $\frac{5}{4} + \frac{5}{8}\sqrt{2}$ |
| m) Diverges | n) Diverges | o) π | |
| 3. $A = 4, 4\ln 3 - 3\ln 5$ | | | |