MATHEMATICS 201-203-RE
Integral Calculus
Martin Huard
Winter 2009

VII - Trigonometric Integrals

1. Evaluate the integral.
   a) $\int \cos^5 x \sin x \, dx$
   b) $\int \cos^3 x \sin^6 x \, dx$
   c) $\int 3\sin^2(7x) \, dx$
   d) $\int \cos^5 \theta \, d\theta$
   e) $\int \sin^2(5t)\cos^2(5t) \, dt$
   f) $\int \sqrt[3]{\sin x} \cos^3 x \, dx$
   g) $\int \frac{\sin^{\frac{3}{2}} x \cos^2 x + \sin x \cos x}{\sqrt{\sin x}} dx$
   h) $\int \cos^4 x \, dx$
   i) $\int \frac{2}{\cos^2(3x-1)} dx$
   j) $\int \tan^3 x \, dx$
   k) $\int \tan^{\frac{2}{3}} x \sec^2 x \, dx$
   l) $\int \tan^3 t \sec^3 t \, dt$
   m) $\int \sec^5 x \tan^3 x \, dx$
   n) $\int \tan^2 x \sec x \, dx$
   o) $\int \frac{\sec x \tan x}{1+\sec^2 x} dx$
   p) $\int \cot^2 \theta \csc^4 \theta \, dx$
   q) $\int \csc 3x \, dx$
   r) $\int \csc^3 x \, dx$
   s) $\int \sin 3x \cos 5x \, dx$
   t) $\int \cos 7\theta \cos 5\theta \, d\theta$
   u) $\int \sin 6x \sin 3x \, dx$

2. Evaluate the given integral.
   a) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos^3 x}{\sqrt{\sin x}} \, dx$
   b) $\int_0^{\frac{\pi}{6}} \sin^2 3x \cos^5 3x \, dx$
   c) $\int_0^{\frac{\pi}{2}} \sin^4 x \, dx$
   d) $\int_{-\frac{\pi}{2}}^{0} \cos^3 x \sin x \, dx$
   e) $\int_{\frac{\pi}{9}}^{\frac{\pi}{6}} \cot 3x \, dx$
   f) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^4 x \, dx$
   g) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^5 x \, dx$
   h) $\int_{-\pi}^{\pi} \sin 3\theta \cos \theta \, d\theta$
   i) $\int_0^{\frac{\pi}{4}} \sin 5\theta \sin \theta \, d\theta$

3. Evaluate the given integral by first using the given substitution.
   a) $\int_0^1 \frac{1}{\sqrt{1+x^2}} dx \qquad x = \tan\theta$

      Hint: Use $1+\tan^2\theta = \sec^2\theta$ to simplify.

   b) $\int \frac{x^2}{\sqrt{1-x^2}} dx \qquad x = \sin\theta$

      Hint: Use $\sin^2\theta + \cos^2\theta = 1$ and $\sin 2\theta = 2\sin\theta\cos\theta$

   c) $\int \frac{1}{\left(1+x^2\right)^2} dx \qquad x = \tan\theta$

Answers

1. a) $-\frac{1}{6}\cos^6 x + C$ or $\frac{1}{2}\sin^2 x - \frac{1}{2}\sin^4 x + \frac{1}{6}\sin^6 x + C$ c
   b) $\frac{1}{7}\sin^7 x - \frac{1}{9}\sin^9 x + C$
   c) $\frac{3}{2}x - \frac{3}{28}\sin(14x) + C$
   d) $\sin\theta - \frac{2}{3}\sin^3\theta + \frac{1}{5}\sin^5\theta + C$
   e) $\frac{1}{8}x - \frac{1}{160}\sin(20x) + C$
   f) $\frac{3}{4}\sin^{\frac{4}{3}} x - \frac{3}{10}\sin^{\frac{10}{3}} x + C$
   g) $\frac{2}{3}\sin^{\frac{3}{2}} x - \frac{1}{3}\cos^3 x + C$
   h) $\frac{3}{8}x + \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x) + C$
   i) $\frac{2}{3}\tan(3x-1) + C$
   j) $\frac{1}{2}\tan^2 x + \ln|\cos x| + C$ or $\frac{1}{2}\tan^2 x - \ln|\sec x| + C$
   k) $\frac{5}{3}\tan^{\frac{3}{5}} x + C$
   l) $\frac{1}{5}\sec^5 t - \frac{1}{3}\sec^3 t + C$
   m) $\frac{1}{7}\sec^7 x - \frac{1}{5}\sec^5 x + C$
   n) $\frac{1}{2}\sec x\tan x - \frac{1}{2}\ln|\sec x + \tan x| + C$
   o) $\arctan(\sec x) + C$
   p) $-\frac{1}{5}\cot^5\theta - \frac{1}{3}\cot^3\theta + C$
   q) $\frac{1}{3}\ln|\csc 3x - \cot 3x| + C$
   r) $-\frac{1}{2}\csc x\cot x + \frac{1}{2}\ln|\csc x - \cot x| + C$
   s) $-\frac{1}{16}\cos 8x + \frac{1}{4}\cos 2x + C$
   t) $\frac{1}{4}\sin 2\theta + \frac{1}{24}\sin 12\theta + C$
   u) $\frac{1}{6}\sin 3x - \frac{1}{18}\sin 9x + C$
2. a) $\frac{8}{5} - \frac{19\sqrt{2}}{20}$
   b) $\frac{8}{315}$
   c) $\frac{3\pi}{16}$
   d) $-\frac{1}{4}$
   e) $\frac{1}{3}\ln 2 - \frac{1}{6}\ln 3$
   f) $\frac{\pi}{4} - \frac{2}{3}$
   g) $\frac{1}{2}\ln 2 - \frac{1}{4}$
   h) 0
   i) $\frac{1}{12}$
3. a) $\ln\left(\sqrt{2}+1\right)$
   b) $\frac{1}{2}\arcsin x - \frac{1}{2}x\sqrt{1-x^2} + C$
   c) $\dfrac{x}{2\left(1+x^2\right)} + \frac{1}{2}\arctan x + C$