

MATHEMATICS 201-203-RE

Integral Calculus

Martin Huard

Winter 2009

V – Substitution (*u- Subs*)

1. Evaluate the indefinite integral.

a) $\int x(x^2 + 1)^{23} dx$

b) $\int \frac{x}{(4x^2 + 9)^2} dx$

c) $\int \frac{3}{\sqrt{4x+1}} dx$

d) $\int x\sqrt{x^2 - 1} dx$

e) $\int \frac{1}{\sqrt{x}} \sin \sqrt{x} dx$

f) $\int \sec^2(3x+5) dx$

g) $\int e^{2x} \sqrt{1+e^{2x}} dx$

h) $\int \frac{\sin 7x}{3 + \cos 7x} dx$

i) $\int \cos x e^{\sin x} dx$

j) $\int \frac{\sin(\frac{3}{x})}{x^2} dx$

k) $\int \frac{1}{x \ln x} dx$

l) $\int \sec^3 x \tan x dx$

m) $\int \frac{1+x}{1+x^2} dx$

n) $\int \frac{(\ln x)^2}{x} dx$

o) $\int \sqrt{e^x} dx$

p) $\int \frac{4x+6}{\sqrt{x^2+3x+1}} dx$

q) $\int x \sin^5(x^2) \cos(x^2) dx$

r) $\int \frac{\sec^2 x}{\sqrt{1+\tan x}} dx$

2. Evaluate the definite integral.

a) $\int_0^2 (x+1)^5 dx$

b) $\int_0^1 e^{2x-1} dx$

c) $\int_0^4 \frac{1}{3x+4} dx$

d) $\int_1^3 \frac{x}{\sqrt{x^2+16}} dx$

e) $\int_0^{\frac{\pi}{4}} \sin 4x dx$

f) $\int_3^6 \frac{1}{(x-2)^3} dx$

g) $\int_0^1 x e^{-x^2} dx$

h) $\int_1^3 \frac{1}{x^2} \sqrt{1+\frac{1}{x}} dx$

i) $\int_0^{13} \frac{1}{\sqrt[3]{(1+2x)^2}} dx$

j) $\int_0^e \frac{1}{x+e} dx$

k) $\int_1^{\sqrt{2}} \frac{x}{1+x^4} dx$

l) $\int_0^{\sqrt{2}} \frac{4}{2+x^2} dx$

m) $\int_1^8 x\sqrt{3x+1} dx$

n) $\int_4^{16} \frac{1}{\sqrt{x}(1+\sqrt{x})^2} dx$

o) $\int_{-1}^{11} \frac{3x-1}{\sqrt{2x+3}} dx$

3. Maintenance costs for buildings increase as the buildings age. If the rate of increase in maintenance costs for a building is $M'(t) = \frac{14000}{\sqrt{2t+9}}$ where M is in dollars and t is time in years, $0 \leq t \leq 15$, find the total maintenance cost for the first 8 years.

4. A piece of machinery purchased for \$100 000 is depreciating at a rate given by $V'(t) = -500e^{-0.20t}$, where t is in years from date of purchase. How much will the machine be worth in five years?
5. After an advertising campaign ends, the rate of change of sales (in thousands) of a product is given by $4te^{-t^2}$, where t is in years. How many will be sold in the next two years?
6. Find the average value of the function on the given interval.

a) $f(x) = x^4\sqrt{2x^5 - 2}$ on $[1, 3]$

b) $f(x) = \frac{\sin 3x}{(2 - \cos 3x)^3}$ on $[\frac{\pi}{3}, \frac{\pi}{2}]$

Answers

1. a) $\frac{1}{48}(x^2 + 1)^{24} + C$ b) $\frac{-1}{8(4x^2 + 9)} + C$ c) $\frac{3}{2}\sqrt{4x+1} + C$
- d) $\frac{1}{3}(x^2 - 1)^{\frac{3}{2}} + C$ e) $-2\cos\sqrt{x} + C$ f) $\frac{1}{3}\tan(3x+5) + C$
- g) $\frac{1}{3}(1 + e^{2x})^{\frac{3}{2}} + C$ h) $-\frac{1}{7}\ln(3 + \cos 7x) + C$ i) $e^{\sin x} + C$
- j) $\frac{1}{3}\cos(\frac{3}{x}) + C$ k) $\ln|\ln x| + C$ l) $\frac{1}{3}\sec^3 x + C$
- m) $\arctan x + \frac{1}{2}\ln(1 + x^2) + C$ n) $\frac{1}{3}(\ln x)^3 + C$ o) $2e^{\frac{1}{2}x} + C$
- p) $4\sqrt{x^2 + 3x + 1} + C$ q) $\frac{1}{12}\sin^6(x^2) + C$ r) $2\sqrt{1 + \tan x} + C$
2. a) $\frac{364}{3}$ b) $\frac{e}{2} - \frac{1}{2e}$ c) $\frac{2}{3}\ln 2$ d) $5 - \sqrt{17}$ e) $\frac{1}{2}$
- f) $\frac{15}{32}$ g) $\frac{1}{2} - \frac{1}{2e}$ h) $\frac{4}{3}\sqrt{2} - \frac{16}{27}\sqrt{3}$ i) 3 j) $\ln 2$
- k) $\frac{1}{2}\arctan 2 - \frac{\pi}{8}$ l) $\frac{\pi\sqrt{2}}{2}$ m) $\frac{644}{5}$ n) $\frac{4}{15}$ o) 40
3. \$28 000
4. \$98 419.70
5. 1963
6. a) $\frac{5324}{15}$ b) $\frac{-5}{36\pi}$