

MATHEMATICS 201-203-RE

Integral Calculus

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Strategy for Integration

Answers

State the method of integration used to evaluate the following integrals then evaluate it.

- $\int \cos^3 3x \, dx = \frac{1}{3} \sin 3x - \frac{1}{9} \sin^3 3x + C$ Trigonometric integral. Let $u = \sin 3x$
- $\int \frac{1}{\sqrt{4 + \sqrt{x+1}}} \, dx = \frac{4}{3} (4 + \sqrt{x+1})^{\frac{3}{2}} - 16\sqrt{4 + \sqrt{x+1}} + C$ v -subs, $v = \sqrt{4 + \sqrt{x+1}}$
- $\int \frac{1}{x^3(1+x)} \, dx = \ln|x| + \frac{1}{x} - \frac{1}{2x^2} - \ln|1+x| + C$ Partial Fractions
- $\int x^2 e^{5x} \, dx = \frac{1}{5} x^2 e^{5x} - \frac{2}{25} x e^{5x} + \frac{2}{125} e^{5x} + C$ By parts (twice) with $u = x^2$ and $dv = e^{5x} \, dx$
- $\int e^x \sin 2x \, dx = \frac{1}{5} e^x \sin 2x - \frac{2}{5} \cos 2x + C$ By parts (twice with a bring over)
- $\int \csc^4(3-2x) \, dx = \frac{1}{2} \cot(3-2x) + \frac{1}{6} \cot^3(3-2x) + C$ Trig integral, $u = \cot(3-2x)$
- $\int \frac{x^3 - 2}{x^2 + 2x + 26} \, dx = \frac{1}{2} x^2 - 2x - 11 \ln|x^2 + 2x + 26| + \frac{72}{5} \arctan \frac{x+1}{5} + C$
Long Division + Complete the Square
- $\int \frac{e^{2x}}{\sqrt[4]{e^x + 2}} \, dx = \frac{4}{7} (e^x + 2)^{\frac{7}{4}} - \frac{8}{3} (e^x + 2)^{\frac{3}{4}} + C$ u -subs, $u = e^x + 2$
- $\int \sin^{\frac{2}{3}} x \cos^3 x \, dx = 3 \sin^{\frac{1}{3}} x - \frac{3}{7} \sin^{\frac{7}{3}} x + C$ Trigonometric integral. Let $u = \sin 3x$
- $\int \ln(x^2 + 16) \, dx = x \ln(x^2 + 16) - 2x + 8 \arctan \frac{x}{4} + C$ By parts, $u = \ln(x^2 + 16)$, and LD
- $\int \frac{2x-17}{x^3 + 8x^2 + 17x} \, dx = -\ln|x| + \frac{1}{2} \ln|x^2 + 8x + 17| + 6 \arctan(x+4) + C$ Partial Fractions + CS
- $\int \frac{x+1}{x\sqrt{x+4}} \, dx = 2\sqrt{x+4} + \frac{1}{2} \ln(\sqrt{x+4}-2) - \frac{1}{2} \ln(\sqrt{x+4}+2) + C$ v -sub $v = \sqrt{x+4}$
- $\int \sin^2(1+x) \, dx = \frac{1}{2} x - \frac{1}{4} \sin(2+2x) + C$ Trig integral. Use $\sin^2(1+x) = \frac{1}{2} - \frac{1}{2} \cos(2+2x)$
- $\int (\sec x - \tan x)^2 \, dx = 2 \tan x - 2 \sec x - x + C$ Expand and use $\tan^2 x = \sec^2 x - 1$
- $\int \frac{x^4 + 4x^3 + 6x^2 + 4x - 3}{x^4 - 1} \, dx = x + 3 \ln|x-1| + \ln|x+1| + 4 \arctan x + C$ LD + partial fractions
- $\int x \arctan 2x \, dx = \frac{1}{2} x^2 \arctan 2x - \frac{1}{4} x + \frac{1}{8} \arctan 2x + C$ By parts, $u = \arctan(2x)$ then LD
- $\int \frac{\cos x}{\sin^4 x} \, dx = \frac{-1}{3 \sin^3 x} + C$ u -sub $u = \sin x$

$$18. \int x^3 \cos x \, dx = x^3 \sin x + 3x^2 \cos x - 6x \sin x - 6 \cos x + C$$

By parts with $u = x^3$ (three times)

$$19. \int \sin 8x \sin 3x \, dx = \frac{1}{10} \sin 5x - \frac{1}{22} \sin 11x + C \quad \text{Trig identity}$$

$$20. \int \frac{3x^2 - x + 1}{(x^2 - x)(x + 1)} \, dx = \frac{3}{2} \ln|x - 1| - \frac{5}{2} \ln|x + 1| - \ln|x| + C \quad \text{Partial Fractions}$$

$$21. \int \tan^4(2x + 1) \, dx = \frac{1}{6} \tan^3(2x + 1) - \frac{1}{2} \tan(2x + 1) + x + C \quad \text{Trig identity } \tan^2 \theta = \sec^2 \theta - 1$$

$$22. \int \frac{x^2 - 4x - 4}{(x - 2)(x^2 + 4)} \, dx = \ln|x^2 + 4| - \ln|x - 2| + C \quad \text{Partial Fractions}$$

$$23. \int \frac{x}{\sqrt[4]{1 + 2x}} \, dx = \frac{1}{7} (1 + 2x)^{\frac{7}{4}} - \frac{1}{3} (1 + 2x)^{\frac{3}{4}} + C \quad v\text{-sub with } v^4 = 1 + 2x$$

$$24. \int \tan^5 x \, dx = \frac{1}{4} \sec^4 x - \sec^2 x + \ln|\sec x| + C \quad \text{Trig integral, let } u = \sec x$$

$$25. \int \frac{x^5}{(x^2 + 1)^2} \, dx = \frac{1}{2} x^2 - \frac{1}{2(x^2 + 1)} - \ln(x^2 + 1) + C \quad \text{LD + Partial Fractions}$$

$$26. \int \frac{1}{x + \sqrt[3]{x}} \, dx = \frac{3}{2} \ln\left(1 + x^{\frac{2}{3}}\right) + C \quad v\text{-sub } v^3 = x$$

$$27. \int \frac{x}{(x - 1)^{\frac{3}{2}}} \, dx = 2\sqrt{x - 1} - \frac{2}{\sqrt{x - 1}} + C \quad v\text{-sub } v^2 = x - 1$$

$$28. \int \frac{x - 2}{\sqrt{x^2 - 4x + 5}} \, dx = \sqrt{x^2 - 4x + 5} + C \quad u\text{-sub } u = x^2 - 4x + 5$$

$$29. \int \frac{4x + 5}{x^2 + 3x + 9} \, dx = 2 \ln|x^2 + 3x + 9| - \frac{2\sqrt{3}}{9} \arctan\left(\frac{2\sqrt{3}}{9}x + \frac{\sqrt{3}}{3}\right) + C \quad \text{Complete the square}$$

$$30. \int \frac{x^2 + 1}{(x + 2)(x^2 + x + 1)} \, dx = \frac{5}{3} \ln|x + 2| - \frac{1}{3} \ln|x^2 + x + 1| + C \quad \text{Partial Fractions + CS}$$