

QUIZ SOLUTIONS

Only the Sharp EL531 calculator is permitted.

Question 1 (6 points)

Using the definition, find the Taylor series for $f(x) = \frac{1}{5x-9}$ at $x = 2$.

$$f(x) = \frac{1}{5x-9} = 5(5x-9)^{-1}$$

$$f(2) = 1$$

$$f'(x) = -5^2(5x-9)^{-2}$$

$$f'(2) = -5$$

$$f''(x) = 5^3 \cdot 2(5x-9)^{-3}$$

$$f''(2) = 5^2 \cdot 2$$

$$f'''(x) = -5^4 \cdot 2 \cdot 3(5x-9)^{-4}$$

$$f'''(2) = -5^3 \cdot 2 \cdot 3$$

$$f^{iv}(x) = 5^5 \cdot 2 \cdot 3 \cdot 4(5x-9)^{-5}$$

$$f^{iv}(2) = 5^4 \cdot 2 \cdot 3 \cdot 4$$

$$f^v(x) = -5^6 \cdot 2 \cdot 3 \cdot 4 \cdot 5(5x-9)^{-6}$$

$$f^v(2) = -5^5 \cdot 2 \cdot 3 \cdot 4 \cdot 5$$

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots$$

$$\frac{1}{5x-9} = 1 - 5(x-2) + \frac{5^2 \cdot 2(x-2)^2}{2!} - \frac{5^3 \cdot 2 \cdot 3(x-2)^3}{3!} + \frac{5^4 \cdot 2 \cdot 3 \cdot 4(x-2)^4}{4!} - \dots$$

$$= 1 - 5(x-2) + 5^2(x-2)^2 - 5^3(x-2)^3 + 5^4(x-2)^4 - \dots$$

$$= \sum_{n=0}^{\infty} (-1)^n 5^n (x-2)^n$$