

MATHEMATICS 201-203-RE

Integral Calculus

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Properties of Sums and Integrals

Properties of Finite Sums

- $$1. \sum_{i=m}^n ca_i = c \sum_{i=m}^n a_i$$
- $$2. \sum_{i=m}^n (a_i \pm b_i) = \sum_{i=m}^n a_i \pm \sum_{i=m}^n b_i$$

Special Sums

- $$1. \sum_{i=1}^n 1 = n$$
- $$2. \sum_{i=1}^n c = cn$$
- $$3. \sum_{i=1}^n i = \frac{n(n+1)}{2}$$
- $$4. \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$
- $$5. \sum_{i=1}^n i^3 = \left[\frac{n(n+1)}{2} \right]^2$$
- $$6. \sum_{i=1}^n i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

The Definite Integral

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \left(\sum_{i=1}^n f(x_i^*) \Delta x \right) \quad \text{where } \Delta x = \frac{b-a}{n}$$

and $x_i^* \in [x_{i-1}, x_i]$ with $x_i = a + i\Delta x$

The Fundamental Theorem of Calculus

Suppose f is continuous on $[a, b]$.

1. If $g(x) = \int_a^x f(t) dt$, then $g'(x) = f(x)$.
2. $\int_a^b f(x) dx = F(b) - F(a)$, where F is any antiderivative of f , that is, $F' = f$.

Properties of the integral

1. $\int cf(x) dx = c \int f(x) dx$
2. $\int [f(x) \pm g(x)] dx = \int f(x) dx \pm \int g(x) dx$
3. $\int_a^b f(x) dx = -\int_b^a f(x) dx$
4. $\int_a^c f(x) dx + \int_c^b f(x) dx = \int_a^b f(x) dx$
5. If $f(x) \geq 0$ for $a \leq x \leq b$, then $\int_a^b f(x) dx \geq 0$.
6. If $f(x) \geq g(x)$ for $a \leq x \leq b$, then $\int_a^b f(x) dx \geq \int_a^b g(x) dx$.
7. If $m \leq f(x) \leq M$ for $a \leq x \leq b$, then $m(b-a) \leq \int_a^b f(x) dx \leq M(b-a)$.
8. $\int u dv = uv - \int v du$ $\int_a^b u dv = uv \Big|_a^b - \int_a^b v du$ (*Integration by parts*)

Table of Indefinite Integrals

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|--------------------------------------------------------|------------------------------------------------------|
| 1. $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ $n \neq -1$ | 2. $\int \frac{1}{x} dx = \ln x + C$ |
| 3. $\int e^x dx = e^x + C$ | 4. $\int a^x dx = \frac{a^x}{\ln a} + C$ |
| 5. $\int \sin x dx = -\cos x + C$ | 6. $\int \cos x dx = \sin x + C$ |
| 7. $\int \sec^2 x dx = \tan x + C$ | 8. $\int \csc^2 x dx = -\cot x + C$ |
| 9. $\int \sec x \tan x dx = \sec x + C$ | 10. $\int \csc x \cot x dx = -\csc x + C$ |
| 11. $\int \sec x dx = \ln \sec x + \tan x + C$ | 12. $\int \csc x dx = \ln \csc x - \cot x + C$ |
| 13. $\int \tan x dx = \ln \sec x + C$ | 14. $\int \cot x dx = \ln \sin x + C$ |
| 15. $\int \frac{1}{1+x^2} dx = \arctan x + C$ | 16. $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$ |