MATHEMATICS 201-203-RE
Integral Calculus
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Integration Techniques with Maple

The different techniques seen here are found in the student package, so the first thing to do is to load the package.

```
with(student);
```

Substitutions

These are done using the **changevar** command which has the following format:

$$\text{changevar}\left(\underbrace{h(u)=g(x)}_{\text{Your substituion}},\underbrace{\text{Int}\left(f(x),x\right)}_{\text{The integral to change}},\underbrace{\qquad u\qquad}_{\text{New variable of integration}}\right)$$

For example, let us evaluate $\int \cos^4 x \sin x\, dx$ using the *u*-sub $u=\cos\theta$.

```
Int(cos(x)^4*sin(x), x ) = changevar(u=cos(x), Int(cos(x)^4*sin(x), x) , u);
NI:=value( changevar(u=cos(x), Int(cos(x)^4*sin(x), x) , u) );
```

To back-substitute, we use the **subs** command,

```
subs(u=cos(x), NI);
```

Note that in the subs command, you need to solve for the variable (in this case *u*).

Integration by parts

For integration by parts, we use the **intparts** command from the student package. The command has the following format:

$$\text{intparts}\left(\underbrace{\text{Int}\left(f(x),x\right)}_{\text{The integral to evaluate}},\underbrace{\qquad u\qquad}_{\text{New expression you let u be}}\right)$$

For example, suppose we wish to evaluate $\int \sqrt{x}\ln x\, dx$, where we let $u=\ln x$ (hence $dv=\sqrt{x}\,dx$).

```
Int(sqrt(x)*ln(x), x) = intparts( Int(sqrt(x)*ln(x), x) , ln(x) );
value( intparts( Int(sqrt(x)*ln(x), x) , ln(x) ) );
```

Partial Fractions

Maple has a command that will convert a rational function to its expression in terms of partial fractions. It uses the keyword **parfrac** in the **convert** command.

For example, suppose we wish to evaluate $\int \frac{5x^2+2x-9}{2x^3-5x^2-3x}dx$.

To convert to partial fractions we have

```
fpar:=convert( (5*x^2+2*x-9)/(2*x^3-5*x^2-3*x), parfrac, x);
```

Evaluating the integral,

```
Int(fpar,x);  value(%);
```

Verifying Answers with Maple

Since there is often more then one way to find an indefinite integral, it may happen that the answer you obtain by doing the techniques seen in class is different then the one obtained with Maple. For example, if we have $\int \cos^3 x\,dx$, then with the methods seen in class we have

$$\begin{aligned}\int \cos^3 x\,dx &= \int \left(1-\sin^2 x\right)\cos x\,dx \\ &= \int \left(1-u^2\right)du \\ &= u - \tfrac{1}{3}u^3 + C \\ &= \sin x - \tfrac{1}{3}\sin^3 x + C\end{aligned}$$

With Maple, we have

```
Int(cos(x)^3,x)=int(cos(x)^3,x);
```

and obtain an answer of $\frac{1}{3}\cos^2 x\sin x + \frac{2}{3}\sin x$.

It can be shown, through the use of trigonometric identities, that the two expressions are equivalent, modulo a constant. To verify integrals with Maple we have the following methods:

1. Try direct integration.
2. Try differentiating your answer, and see if it gives the original expression.
```
diff(sin(x)-sin(x)^3/3,x);
simplify(%);
```
3. Use the same methods that you used but with Maple.
```
Int(cos(x)^3,x)= changevar(u=sin(x), Int(cos(x)^3, x), u);
NI:=value( changevar(u=sin(x), Int(cos(x)^3, x), u) );
subs(u=sin(x), NI);
```
4. Graph the difference in the two answers. Since they should only differ by a constant, if your answer is correct, the plot should be a horizontal line.
```
plot( int(cos(x)^3,x) - (sin(x)-1/3*sin(x)^3) ,x=-20..20);
```

Note that the plot you obtain is not a straight line, even if algebraically the difference between the two functions is exactly zero. What you are actually seeing are the rounding off errors in the evaluation of the trigonometric functions. Take a look at the scale on the *y*-axis, it goes from -10^{-16} to 10^{-16}. By default, Maple works with an accuracy of 10 decimals, so we are far below this. Also, this graphical method does not actually prove that you have the right answer, although it can certainly serves as an indication, especially if you are wrong.

Tutor

If you don't know how to proceed to integrate a function, the use of Maple's Tutor may prove helpful. For example, try $\int \cos^3 x\,dx$. In Maple go to TOOLS → TUTORS → CALCULUS SINGLE VARIABLE → INTEGRATION METHODS. Enter the function in the appropriate place and go through the steps.