

## MATHEMATICS 201-203-RE

Integral Calculus

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# IV - The Fundamental Theorem of Calculus

1. Use Part I of the Fundamental Theorem of Calculus to find the derivative of the function.

a)  $g(x) = \int_1^x \cos \sqrt{t} \, dt$

b)  $g(x) = \int_4^x \frac{3du}{1+\sqrt[5]{u}}$

c)  $g(x) = \int_{-4}^{x^3} e^{t^2} \, dt$

d)  $g(x) = \int_x^3 \sin(t^2) \, dt$

e)  $g(x) = \int_{\frac{1}{x}}^{x^4} \frac{1}{\cos t} \, dt$

f)  $g(x) = \int_{\sec x}^{3x^2} \sqrt{1+t^2} \, dt$

2. Evaluate the definite integral.

a)  $\int_1^3 x^5 \, dx$

b)  $\int_{-3}^0 (x^2 - 5x + 1) \, dx$

c)  $\int_1^4 \frac{1}{x^2} \, dx$

d)  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin t \, dt$

e)  $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos \theta \, d\theta$

f)  $\int_4^9 x\sqrt{x} \, dx$

g)  $\int_1^4 \left( 3\sqrt{t} - \frac{4}{\sqrt{t}} + \sqrt[3]{t^2} \right) dt$

h)  $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left( x + \frac{2}{\sin^2 x} \right) dt$

i)  $\int_1^{\sqrt{3}} \frac{2}{1+x^2} \, dx$

j)  $\int_1^4 \frac{x^3 - 1}{\sqrt{x}} \, dx$

k)  $\int_0^2 (x+1)^3 \, dx$

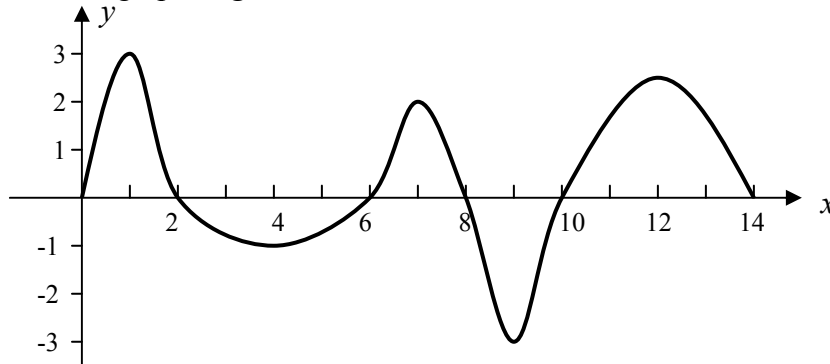
l)  $\int_1^3 \left( e^x + \sqrt{\frac{5}{x}} \right) dx$

m)  $\int_{-1}^6 f(x) \, dx$  where  $f(x) = \begin{cases} 3x^2 - x & x \leq 1 \\ \frac{x+1}{x} & x > 1 \end{cases}$

n)  $\int_{-1}^4 |2x-4| \, dx$

3. Let  $g(x) = \int_0^x f(t) \, dt$ , where  $f$  is the function whose graph is given below.

- Find the values of  $x$  for which  $g$  has local maximum's and minimum's.
- Find the values of  $x$  for where  $g$  has a point of inflection.
- Sketch the graph of  $g$ .



4. Evaluate the following limit by first recognizing the sum as a Riemann sum for a function defined on the interval  $[0,1]$ .

$$\text{a) } \lim_{n \rightarrow \infty} \left( \frac{1}{n^3} + \frac{4}{n^3} + \frac{9}{n^3} + \cdots + \frac{n^2}{n^3} \right)$$

$$\text{b) } \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{n}{n^2 + i^2}$$

$$\text{c) } \lim_{n \rightarrow \infty} \frac{1}{n^2} \left( \sqrt{n} + \sqrt{2n} + \sqrt{3n} + \sqrt{4n} + \cdots + \sqrt{n^2} \right)$$

5. Find the average value of the function on the given interval.

$$\text{a) } f(x) = 4x^3 - 3x^2 + 2 \text{ on } [-3,1]$$

$$\text{b) } f(x) = \cos x \text{ on } \left[ -\frac{\pi}{4}, \frac{\pi}{4} \right]$$

$$\text{c) } f(x) = e^x \text{ on } [1,4]$$

6. Find a value of  $c$  that satisfies the conclusion to the mean value theorem for integrals.

$$\text{a) } f(x) = 4x^3 - 2 \text{ on } [1,3]$$

$$\text{b) } f(x) = \frac{1}{1+x^2} \text{ on } [0,1]$$

7. The number of daily sales of a product was found to be given by

$$S = 100\left(\frac{1}{2}\right)^x + 100$$

$x$  days after the start of an advertising campaign for this product. Find the average daily sales during the first 10 days of the campaign.

8. If the demand function for a product is given by

$$p = 500 + \frac{1000}{q+1}$$

where  $p$  is the price and  $q$  is the number of units demanded, find the average price as demand ranges from 49 to 99 units.

9. The manager of a mathematics magazine estimates that the total number of subscribers  $t$  years from now will be

$$N = 50\,000 - \frac{40\,000}{1+t^2}$$

Find the average number of subscribers over the next 5 years if this prediction holds true.

10. Suppose that water can flow in and out of a storage tank. The net rate of change (that is the rate in minus the rate out) of water is  $f'(t) = 20(t^2 - 1)$  liters per minute.

a) For  $0 \leq t \leq 3$ , find the change in the amount of water in the reservoir.

b) If the tank has 200 liters of water at time  $t = 0$ , determine how many liters are in the tank at  $t = 3$ .

11. Pollution from a factory is entering a lake. The rate of concentration of the pollutant at time  $t$  is given by

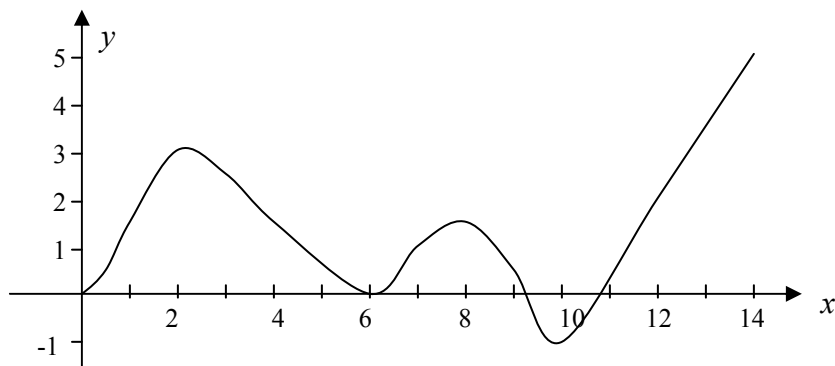
$$P'(t) = 140t^{\frac{5}{2}}$$

where  $t$  is the number of years since the factory started introducing pollutants into the lake. Ecologists estimate that the lake can accept a total level of pollution of 4850 units before all the fish life in the lake ends. Can the factory operate for 4 years without killing all the fish in the lake?

12. Suppose oil is being extracted from a field at a rate given by  $P'(t) = 0.2 \cdot (1.105)^t$ , where  $P(t)$  is measured in millions of barrels and  $t$  in years. At this rate how much oil will be extracted during the second 10-year period?
13. Suppose copper is being extracted from a certain mine at a rate given by  $\frac{d}{dt}P(t) = 100 \cdot (0.8187)^t$ , where  $P(t)$  is measured in tons of copper and  $t$  in years. At this rate how much copper will be extracted during the third year?
14. A firm estimates that its sales (in millions of dollars) will increase at a rate given by  $\frac{d}{dt}S(t) = 10\sqrt{t}$ , where  $t$  is in years. Find the total sales during the second and third year.
15. A company purchases a new machine for which the rate of depreciation can be modeled by  $\frac{d}{dt}V(t) = 10000(t-6)$  where  $V$  is the value of the machine after  $t$  years ( $0 \leq t \leq 5$ ). Find the loss of value of the machine over the first three years.
16. The current circulation of the Personal Computer Secrets magazine is 4000 per week. Circulation is expected to grow at a rate of  $6 + 4t^{\frac{1}{3}}$  copies per week,  $t$  weeks from now, for the next five years. Determine the increase in circulation of the magazine 75 weeks from now. What will be the circulation of the magazine 75 weeks from now?
17. As part of their quality control process at PC's R Us, each computer undergoes a final inspection before packing. The rate of change of the number of PC's that an inspector can check in an hour  $t$  hours into an 8-hour shift is approximately  $-3t^2 + 9t + 60$  ( $0 \leq t \leq 8$ ). Find the total number of computers inspected 6 hours into their shift.
18. A store finds that its sales change at a rate of  $-3t^2 + 300t$  where  $t$  is the number of days after an advertising campaign ends, and  $0 \leq t \leq 30$ . Find the total sales for the second week after the campaign ends.
19. After  $t$  hours on the job, one factory worker is producing  $Q_1(t) = 60 - 2(t-1)^2$  units per hour, while a second worker is producing  $Q_2(t) = 50 - 5t$  units per hour. If both arrive on the job at 8:00 A.M., how many more units will the first worker have produced by noon than the second worker?

## Answers

1. a)  $\cos \sqrt{x}$       b)  $\frac{3}{1+\sqrt[5]{x}}$       c)  $3x^2 e^{x^6}$   
 d)  $-\sin(x^2)$       e)  $\frac{4x^3}{\cos(x^4)} + \frac{1}{x^2 \cos(\frac{1}{x})}$       g)  $6x\sqrt{1+9x^4} - \sec x \tan x \sqrt{1+\sec^2 x}$
2. a)  $\frac{364}{3}$       b)  $\frac{69}{2}$       c)  $\frac{3}{4}$       d) 0      e)  $\sqrt{2}$   
 f)  $\frac{422}{5}$       g)  $\frac{27}{5} + \frac{24}{5} \sqrt[3]{2}$       h)  $\frac{\pi^2}{9} + 2\sqrt{3}$       i)  $\frac{\pi}{6}$       j)  $\frac{240}{7}$   
 k) 20      l)  $e^3 + 2\sqrt{15} - e - 2\sqrt{5}$       m)  $7 + \ln 6$       n) 13
3. a) maximum at  $x = 2, 8$  and 14      minimum at  $x = 6, 10$   
 b) Inflection points at  $x = 1, 4, 7, 9, 12$   
 c)



4. a)  $\frac{1}{3}$       b)  $\frac{\pi}{4}$       c)  $\frac{2}{3}$   
 5. a) -25      b)  $\frac{2\sqrt{2}}{\pi}$       c)  $\frac{1}{3}e^4 - \frac{1}{3}e$   
 6. a)  $\sqrt[3]{10}$       b)  $\frac{\sqrt{4\pi - \pi^2}}{\pi}$   
 7.  $\approx 114$  sales  
 8. \$513.86  
 9. 39 013 subscribers  
 10. a) 120 liters      b) 320 liters  
 11. No. The pollution in the lake will increase by 5120 units in the first four years.  
 12. 9.32 millions of barrels  
 13. 60.7 tons of coppers  
 14. 27.97 millions of dollars  
 15. 135 000 dollars  
 16. a) 1399 copies      b) 5399 copies  
 17. 306 computers  
 18. 19 649 dollars  
 19. 61 units