MATHEMATICS 201-203-RE
Integral Calculus
Martin Huard
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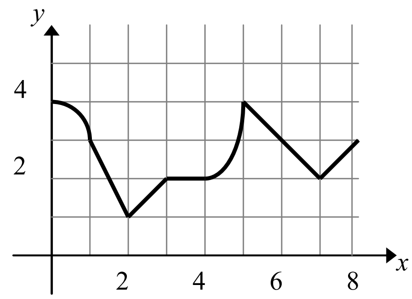
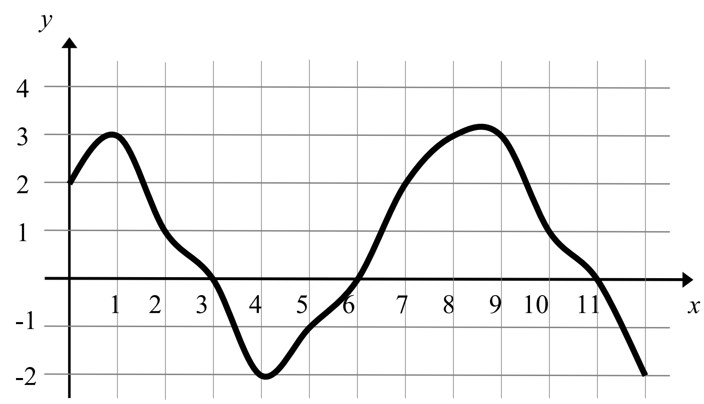
III - The Definite Integral

1. The graph of a function f is given.
   Estimate $\int_0^8 f(x)\,dx$ using four subintervals with
   a) right endpoints
   b) left endpoints
   c) midpoints



2. The graph of a function f is given.
   Estimate $\int_1^{11} f(x)\,dx$ using five subintervals with
   a) right endpoints
   b) left endpoints
   c) midpoints



3. Evaluate each of the following definite integral using a Riemann Sum. (The answers are given using the RHE). Does the answer represent an area? Explain.
   a) $\int_0^2 \left(1-\frac{1}{2}x\right)dx$
   b) $\int_{-2}^1 (2x+1)\,dx$
   c) $\int_{-1}^3 (1+x)(3-x)\,dx$
   d) $\int_0^3 \left(x^2-4x+3\right)dx$
   e) $\int_0^4 \left(2x^2+x-1\right)dx$
   f) $\int_{-3}^{-1} (4x-3)(3x+1)\,dx$
   g) $\int_{-1}^3 \left(-x^2+4x-5\right)dx$
   h) $\int_2^5 \left(x^2+x+1\right)dx$
   i) $\int_{-1}^1 \left(x^3+1\right)dx$
   j) $\int_{-3}^3 (x+2)(x-1)^2\,dx$

4. Prove the following.
   a) $\int_a^b x\,dx = \dfrac{b^2-a^2}{2}$.
   b) $\int_a^b x^2 dx = \dfrac{b^3-a^3}{3}$.

5. If $\int_1^6 f(t)\,dt = 2$ and $\int_3^6 f(t)\,dt = 5$, find $\int_1^3 f(t)\,dt$.

6. If $\int_{-1}^4 f(x)\,dx = 7$ and $\int_{-1}^1 f(x)\,dx = -1$, find $\int_1^4 5f(x)\,dx$.

7. Use the properties of integrals to verify the inequality without evaluating the integral.

$$\tfrac{\pi}{12} \le \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin^2 x\,dx \le \tfrac{\pi}{3}$$

8. Use the properties of integrals (along with question 3) to prove $\int_0^{\frac{\pi}{2}} x\sin x\,dx \le \frac{\pi^2}{8}$.

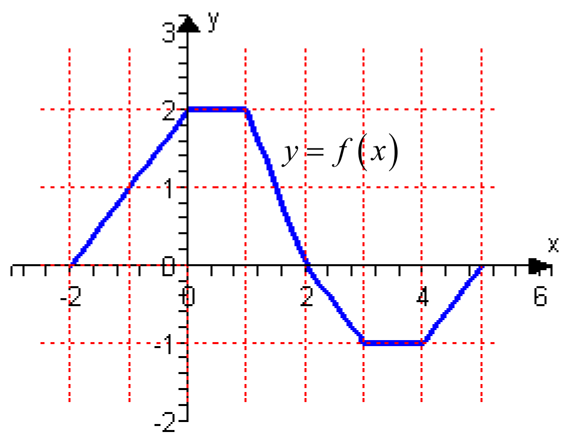
9. Evaluate the integral by interpreting it in terms of area.

a) $\int_{-2}^2 f(x)\,dx$

b) $\int_2^5 f(x)\,dx$

c) $\int_0^4 f(x)\,dx$

d) $\int_{-2}^5 f(x)\,dx$



10. Express the limit as a definite integral.

a) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^6}{n^7}$

b) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3\sqrt{2+\frac{3i}{n}}}{n}$

c) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi \sin \frac{\pi i}{2n}}{2n}$

d) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\sqrt{n^2 - i^2}}{n^2}$

Answers

1. a) 18 b) 20 c) 22
2. a) 8 b) 14 c) 6
3. a) $\lim_{n\to\infty} \frac{(n-1)}{n} = 1$ Yes b) $\lim_{n\to\infty} \frac{9}{n} = 0$ No

   c) $\lim_{n\to\infty} \frac{32(n-1)(n+1)}{3n^2} = \frac{32}{3}$ Yes d) $\lim_{n\to\infty} \frac{-9(n-1)}{2n^2} = 0$ No

   e) $\lim_{n\to\infty} \frac{4(7n+8)(5n+2)}{3n^2} = \frac{140}{3}$ No f) $\lim_{n\to\infty} \frac{2(59n^2-53n+8)}{n^2} = 118$ Yes

   g) $\lim_{n\to\infty} \frac{-8(5n^2-6n+4)}{3n^2} = -\frac{40}{3}$ No h) $\lim_{n\to\infty} \frac{3(35n^2+24n+3)}{2n^2} = \frac{105}{2}$ Yes

   i) $\lim_{n\to\infty} \frac{2(n+1)}{n} = 2$ Yes j) $\lim_{n\to\infty} \frac{12(n+9)}{n} = 12$ No
4. Use the Riemann sum
5. –3
6. 40
7. Use the inequality $\frac{1}{2} \le \sin x \le 1$ for $x \in \left[\frac{\pi}{6}, \frac{\pi}{2}\right]$.
8. Use the inequality $x \sin x \le x$ for $x \in \left[0, \frac{\pi}{2}\right]$
9. a) 5 b) -2 c) $\frac{3}{2}$ d) 3
10. a) $\int_0^1 x^6 dx$ b) $\int_2^5 \sqrt{x} dx$ c) $\int_0^{\frac{\pi}{2}} \sin x \, dx$ d) $\int_0^1 \sqrt{1-x^2} dx$