MATHEMATICS 201-203-RE
Integral Calculus
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I - Antiderivatives

1. Find the antiderivatives.

a) $\int \left(6x^2 - 8x + 3\right) dx$

b) $\int \left(x - \sqrt{x}\right) dx$

c) $\int \left(\frac{1}{x^2} - \frac{1}{x^3}\right) dx$

d) $\int \left(\frac{1}{2x^4} - \frac{6}{\sqrt[4]{x}} + \sqrt{2\pi}\right)$

e) $\int \left(x\sqrt{x} + \sqrt[5]{x^3}\right) dx$

f) $\int \left(\frac{1}{x^2\sqrt{x}} + \frac{x^2}{\sqrt[3]{x}}\right) dx$

g) $\int 2\sqrt{x}\left(3x^2 + 6x^3 - 1\right) dx$

h) $\int \left(1 + x^2\right)\left(2x + 1\right) dx$

i) $\int \frac{3x^3 - 2x + 1}{x^2} dx$

j) $\int \frac{x^3 + 2x^2}{\sqrt{x}} dx$

k) $\int \left(\frac{2}{x} + 3e^x\right) dx$

l) $\int \left(4\sin x - 3\cos x\right) dx$

m) $\int \frac{5}{1 + x^2} dx$

n) $\int \frac{\sin x}{\cos^2 x} dx$

o) $\int \left(\frac{\sec x \tan x + \csc x}{\sin x}\right) dx$

p) $\int \frac{e^{-x} + 1}{e^{-x}} dx$

q) $\int \frac{x - 1}{\sqrt{x} + 1} dx$

r) $\int \frac{x^5 - x^4 + 2x^3 + x^2 + 6}{x^2 + 2} dx$

2. Find $f(x)$.

a) $f''(x) = 24x^2 - 4x + 2$

b) $f''(x) = 5x^4 + \frac{1}{\sqrt{x}}$

c) $f'''(x) = e^x$

d) $f'(x) = 1 + \frac{1}{x^2}$ $\quad f(1) = 2$

e) $f''(x) = 12x^3 + 6x$ $\quad f(1) = 2$ $\quad f'(1) = 5$

f) $f''(x) = x^2 + \sin x$ $\quad f(0) = -1$ $\quad f'(0) = 3$

g) $f''(x) = 18x^2 - 6$ $\quad f(1) = 2$ $\quad f(3) = 5$

3. The weekly marginal cost for producing iPods can be model with $C'(x) = \frac{3}{10000}x^2 - \frac{1}{25}x + 20$, where $C'(x)$ is measured in dollars/unit and x denotes the number of units produced. It has been determined that the weekly fixed cost are \$3000. Find the total cost for producing the first 300 iPods.

4. The weekly marginal revenue for selling iPods can be model with $R'(x) = \frac{2}{25}x + 40$, where $R'(x)$ is measured in dollars/unit and x denotes the number of units produced. Find the demand function. What is the price of iPods if 300 iPods are sold?

5. The marginal profit for a small manufacturer is given by $P(x) = 5x(\sqrt{x} - 10)$, where x is the number of units produced (and sold) per month and P is in dollars. If the fixed costs are 5000 dollars per month, find the profit function.

6. Find the demand function for a manufacturer if marginal demand, in dollars, is given by $p'(x) = -x^{-\frac{3}{2}}$, where x is the number of thousands of units sold. Assume that the price for a thousand unit is \$100.

7. The marginal cost and revenue functions for a given product are
$$C'(x) = \frac{1}{10}\left(\frac{11}{10}\right)^x \qquad R'(x) = \sqrt[3]{x^4}$$
dollars per thousand units. The fixed costs are \$3000.
   a) Find the cost function.
   b) Find the demand function.
   c) What is the price for a production of 64 000 units?
   d) Find the profit function.
   e) What is the profit for a production of 64 000 units?

8. It is estimated that the population of a small suburb is increasing at the rate of $4500\sqrt{t} + 1000$ people/year t years from now. The population before construction is 30 000. Determine the projected population in 9 years.

9. It has been estimated that the rate of change of the number of new cell phones per 1000 Canadian population is given by $T'(x) = \dfrac{250}{x}$ where in 2001 ($x = 1$), there were 200 phones per 1000 population. How many cell phones are there in 2009?

10. The temperature (in ^{o}C) outside on certain January day is changing at a rate of
$$T'(t) = -\tfrac{1}{2}t + 3 \qquad 0 \le t \le 8$$
where t is in hours, and $t = 0$ corresponds to 6 a.m. The temperature at 6 a.m. is -20 ^{o}C.
   a) Find the temperature outside at a given time t.
   b) Find the temperature at 10 a.m.

Answers

1. a) $2x^3-4x^2+3x+C$ b) $\frac{x^2}{2}-\frac{2}{3}x^{\frac{3}{2}}+C$ c) $-\frac{1}{x}+\frac{1}{2x^2}+C$

   d) $\frac{-1}{6x^3}-8x^{\frac{3}{4}}+\sqrt{2\pi}x+C$ e) $\frac{2}{5}x^{\frac{5}{2}}+\frac{5}{8}x^{\frac{8}{5}}+C$ f) $\frac{-2}{3x^{\frac{3}{2}}}+\frac{3}{8}x^{\frac{8}{3}}+C$

   g) $\frac{4}{3}x^{\frac{3}{2}}+\frac{12}{7}x^{\frac{7}{2}}+\frac{8}{3}x^{\frac{9}{2}}+C$ h) $\frac{x^4}{2}+\frac{x^3}{3}+x^2+x+C$ i) $\frac{3}{2}x^2-\frac{1}{x}-2\ln|x|+C$

   j) $\frac{4}{5}x^{\frac{5}{2}}+\frac{2}{7}x^{\frac{7}{2}}+C$ k) $2\ln|x|+3e^x+C$ l) $-4\cos x-3\sin x+C$

   m) $5\arctan x+C$ n) $\sec x+C$ o) $\tan x-\cot x+C$

   p) $x+e^x+C$ q) $\frac{2}{3}x^{\frac{3}{2}}-x+C$ r) $\frac{1}{4}x^4-\frac{1}{3}x^3+3x+C$

2. a) $f(x)=2x^4-\frac{2}{3}x^3+x^2+Cx+D$ b) $f(x)=\frac{1}{6}x^6+\frac{4}{3}x^{\frac{3}{2}}+Cx+D$

   c) $f(x)=e^x+Cx^2+Dx+E$ d) $f(x)=x-\frac{1}{x}+2$

   e) $f(x)=\frac{3}{5}x^5+x^3-x+\frac{7}{5}$ f) $f(x)=\frac{1}{12}x^4-\sin x+4x-1$

   g) $f(x)=\frac{3}{2}x^4-3x^2-\frac{93}{2}x+50$

3. \$9900
4. \$52
5. $P(x)=2x^{\frac{5}{2}}-25x^2-5000$
6. $p(x)=\frac{2}{\sqrt{x}}+98$
7. a) $C(x)=\frac{11^x-10^{x+1}}{10^{x+1}(\ln 11-\ln 10)}+3000$ b) $p(x)=\frac{3}{7}x^{\frac{4}{3}}$

   c) $\frac{768}{7}\approx \$109.71$ d) $P(x)=\frac{3}{6}x^{\frac{7}{3}}-\frac{11^x-10^{x+1}}{10^{x+1}(\ln 11-\ln 10)}-3000$

   e) \$3555.03
8. 120000
9. $250\ln 9+200\simeq 749.3$ per 1000 population
10. a) $T(t)=-\frac{1}{4}t^2+3t-20$ b) -12°C