MATHEMATICS 201-203-RE
Integral Calculus
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Winter 2009

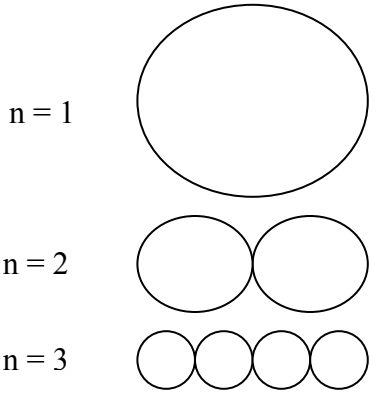
Assignment #4

This assignment is due on ***Monday April 20, 2009*** at the beginning of class.
Complete solutions with exact answers are expected whenever possible.

For questions involving Maple, a print-out of your work is expected, where your name is written in the Worksheet, each question is clearly labeled, and the answers are clearly presented. Also, you must copy your file in my "TEST" subfolder (W:\Tests\mhuard\203\Assignment 4), where your name should be included in the name of the file (for example: Assignment 4 – Your Name).

Question 1 (4 points)

Consider the following sequence of circles. We start with a circle of diameter 2. Next, on the diameter of the first circle, we make two circles. On each of these circles, we again make two circles. We continue this process indefinitely.



a) Let P_n be the perimeter of all the circles after n iteration. Does the sequence P_n converge? If so, to what number?
b) Let A_n be the total area occupied by all the circles after n iteration. Does the sequence A_n converge? If so, to what number?

Question 2 (4 points)

Suppose Gauss just won a lottery. He is offered either a lump sum of 10 million dollars now, or $100 000 a month for life. Assuming money is worth 10% compounded monthly, how many years will it take for the lump sum of 10 million dollars to be equivalent to the $100 000 a month?

Question 3 (5 points)

If *$R* is deposited at the beginning of each period for *n* periods in an account that earns interest at a rate of *i* per period, what will be the balance *S* after *n* periods? This is often referred to as the future value of an **annuity due**.

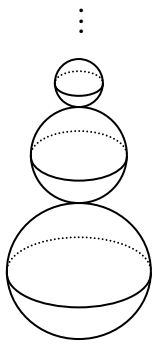
Question 4 (4 points)

Euler would like to have $400 000 saved by the time he retires. Suppose that he is to deposit money in an account paying 10% interest per year compounded monthly. Also, suppose that he is now 20 years old and plans to retire at 60 years of age.

a) If he starts saving now, how much money should he place in the account each month?
b) If he will only start saving 20 years from now, how much money will he have to place in the account each month?

Question 5 (8 points)

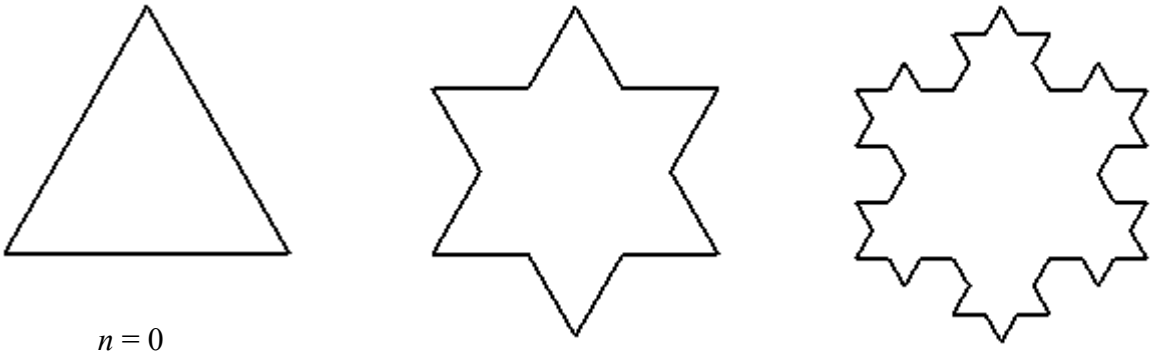
Imagine you are stacking an infinite number of spheres of decreasing radii on top of each other, as indicated in the figure. The radii of the spheres are 1 m, $\frac{1}{\sqrt{2}}$ m, $\frac{1}{\sqrt{3}}$ m, etc.



a) How high is this infinite stack of spheres?
b) What is the total surface area of all the spheres in the stack?
c) Show that the volume of the stack is finite.
d) Approximate the volume of the stack with Maple.

Question 6 (10 points)

A **sequence of Snowflakes**. Start with an equilateral triangle with sides of length 1. The construction is as follow. The first step is to divide each side into three equal parts, construct an equilateral triangle on the middle part, and then delete the middle part. Let us call this snowflake S_1. The second step is to repeat the first step for each side of the resulting polygon to obtain S_2. This process is repeated at each step, obtaining a sequence of snowflakes. The resulting curve is called the **Koch snowflake**, named for Helge von Koch, who described it in 1904.



$n = 0$ $n = 1$ $n = 2$

a) Let s_n, l_n and P_n represent the number of sides, the length of a side and the total length of the nth snowflake. Find formulas for s_n, l_n, and P_n.

b) Evaluate $\lim_{n\to\infty} P_n$.

c) Let a_n and t_n represent the area of the smallest triangle and the number of such triangles added on the nth snowflake. Find formulas for a_n and t_n.

d) Let A_n be the area of the nth snowflake. Evaluate $\lim_{n\to\infty} A_n$. [*Hint*, Express A_n as a sum, with a_0 not included in the summation.]

Question 6 (15 points)

Determine if the following series converge.

a) $\displaystyle\sum_{k=1}^{\infty} \frac{k+1}{e^k}$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{3n-1}{\sqrt[3]{n^8 - n^2 + 4}}$

c) $\displaystyle\sum_{n=0}^{\infty} \frac{3^n n!}{(2n)!}$