MATHEMATICS 201-203-RE
Integral Calculus
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Assignment #2

SOLUTIONS

This assignment is due ***Monday February 23, 2009*** at the beginning of class.

Question 1 (45 points)

Evaluate the following.

a) $$\int \frac{\arctan\sqrt{x}}{\sqrt{x}}\,dx = 2\sqrt{x}\arctan\sqrt{x} - \int \frac{1}{2\sqrt{x}\left(1+x\right)}2\sqrt{x}dx$$

$$u = \arctan\sqrt{x} \qquad v = 2\sqrt{x}$$

$$du = \frac{1}{1+\left(\sqrt{x}\right)^2}\frac{1}{2\sqrt{x}}dx \qquad dv = \frac{1}{\sqrt{x}}dx$$

$$= \frac{1}{2\sqrt{x}\left(1+x\right)}dx$$

$$= 2\sqrt{x}\arctan\sqrt{x} - \int \frac{1}{1+x}dx$$

$$= 2\sqrt{x}\arctan\sqrt{x} - \ln\left|1+x\right| + C$$

b) $$\int \sin 3x\, e^{5x} dx$$

$$\int \sin 3x\, e^{5x} dx = -\tfrac{1}{3}\cos 3x\, e^{5x} + \int \tfrac{5}{3}\cos 3x\, e^{5x} dx \qquad u = e^{5x} \qquad v = -\tfrac{1}{3}\cos 3x$$

$$= -\tfrac{1}{3}\cos 3x\, e^{5x} + \tfrac{5}{3}\left(\tfrac{1}{3}\sin 3x\, e^{5x} - \tfrac{5}{3}\int \sin 3x\, e^{5x} dx\right) \qquad du = 5e^{5x}dx \qquad dv = \sin 3x\, dx$$

$$u = e^{5x} \qquad v = \tfrac{1}{3}\sin 3x$$

$$= -\tfrac{1}{3}\cos 3x\, e^{5x} + \tfrac{5}{9}\sin 3x\, e^{5x} - \tfrac{25}{9}\int \sin 3x\, e^{5x} dx \qquad du = 5e^{5x}dx \qquad dv = \cos 3x\, dx$$

$$\tfrac{34}{9}\int \sin 3x\, e^{5x} dx = -\tfrac{1}{3}\cos 3x\, e^{5x} + \tfrac{5}{9}\sin 3x\, e^{5x}$$

$$\int \sin 3x\, e^{5x} dx = -\tfrac{3}{34}\cos 3x\, e^{5x} + \tfrac{5}{34}\sin 3x\, e^{5x} + C$$

c) $\int \sec^6 x\,dx$

$$\int \sec^6 x\,dx = \int \sec^4 x \sec^2 x\,dx \qquad u = \tan x$$

$$du = \sec^2 x\,dx$$

$$= \int \left(1+\tan^2 x\right)^2 \sec^2 x\,dx$$

$$= \int \left(1+u^2\right)^2 du$$

$$= \int \left(1+2u^2+u^4\right) du$$

$$= u + \tfrac{2}{3}u^3 + \tfrac{1}{5}u^5 + C$$

$$= \tan x + \tfrac{2}{3}\tan^3 x + \tfrac{1}{5}\tan^5 x + C$$

d) $\int \frac{3x^3+9x^2+12x+16}{x^4+4x^2}\,dx$

$$\frac{3x^3+9x^2+12x+16}{x^4+4x^2} = \frac{3x^3+9x^2+12x+16}{x^2\left(x^2+4\right)}$$

$$= \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+4}$$

$$= \frac{Ax\left(x^2+4\right)+B\left(x^2+4\right)+\left(Cx+D\right)x^2}{x^2\left(x^2+4\right)}$$

$$3x^3+9x^2+12x+16 = Ax\left(x^2+4\right)+B\left(x^2+4\right)+\left(Cx+D\right)x^2$$

$$= \left(A+C\right)x^3 + \left(B+D\right)x^2 + 4Ax + 4B$$

$A+C=3 \qquad \Rightarrow C=0$

$B+D=9 \qquad \Rightarrow D=5$

$4A=12 \qquad \Rightarrow A=3$

$4B=16 \qquad \Rightarrow B=4$

$$\int \frac{3x^3+9x^2+12x+16}{x^4+4x^2}\,dx = \int \left(\frac{3}{x}+\frac{4}{x^2}+\frac{5}{x^2+4}\right)dx$$

$$= 3\ln|x| - \frac{4}{x} + \tfrac{5}{2}\arctan\left(\tfrac{x}{2}\right) + C$$

e) $\displaystyle\int \frac{x^4+4x^3+19x^2}{x^2+4x+20}\,dx$

$$\begin{array}{r} x^2-1 \\ x^2+4x+20\overline{\smash{)}x^4+4x^3+19x^2} \\ \underline{x^4+4x^3+20x^2} \\ -x^2 \\ -\underline{x^2-4x-20} \\ 4x+20 \end{array}$$

$$\int \frac{x^4+4x^3+19x^2}{x^2+4x+20}\,dx = \int\left(x^2-1+\frac{4x+20}{x^2+4x+20}\right)dx$$

$$= \tfrac{1}{3}x^3 - x + \int \frac{4x+20}{(x+2)^2+16}\,dx$$

$u = x+2$

$du = dx$

$$= \tfrac{1}{3}x^3 - x + \int \frac{4u+12}{u^2+16}\,du$$

$$= \tfrac{1}{3}x^3 - x + \int \frac{4u}{u^2+16}\,dx + \int \frac{12}{u^2+16}\,dx$$

$v = u^2+16$

$dv = 2u\,du$

$$= \tfrac{1}{3}x^3 - x + \int \frac{2}{v}\,dv + 3\arctan\tfrac{u}{4}$$

$$= \tfrac{1}{3}x^3 - x + 2\ln|v| + 3\arctan\tfrac{u}{4} + C$$

$$= \tfrac{1}{3}x^3 - x + 2\ln\left|u^2+16\right| + 3\arctan\tfrac{u}{4} + C$$

$$= \tfrac{1}{3}x^3 - x + 2\ln\left|x^2+4x+20\right| + 3\arctan\tfrac{x+2}{4} + C$$

f) $\displaystyle\int \frac{\sqrt[8]{x}}{\sqrt[4]{x}-1}\,dx$

$$\int \frac{\sqrt[8]{x}}{\sqrt[4]{x}-1}\,dx = \int \frac{v}{v^2-1}8v^7dv$$

$$v = x^{\frac{1}{8}}$$
$$v^8 = x$$
$$8v^7dv = dx$$

$$= \int \frac{8v^8}{v^2-1}dv$$

$$= \int\left(8v^6+8v^4+8v^2+8+\frac{8}{v^2-1}\right)dv$$

$$\begin{array}{r} v^6+v^4+v^2+1 \\ v^2-1\big)\overline{v^8} \\ \underline{v^8-v^6} \\ v^6 \\ \underline{v^6-v^4} \\ v^4 \\ \underline{v^4-v^2} \\ v^2 \\ \underline{v^2-1} \\ 1 \end{array}$$

$$\frac{8}{v^2-1} = \frac{8}{(v-1)(v+1)}$$
$$= \frac{A}{v-1}+\frac{B}{v+1}$$
$$= \frac{A(v+1)+B(v-1)}{(v-1)(v+1)}$$

$$8 = A(v+1)+B(v-1)$$
$$8 = (A+B)v+A-B$$
$$v: 0 = A+B$$
$$\underline{cst: 8 = A-B}$$
$$8 = 2A \qquad \Rightarrow \qquad A=4, \quad B=-4$$

$$\int \frac{\sqrt[8]{x}}{\sqrt[4]{x}-1}\,dx = \tfrac{8}{7}v^7+\tfrac{8}{5}v^5+\tfrac{8}{3}v^3+8v+\int\left(\frac{4}{v-1}-\frac{4}{v+1}\right)dv$$
$$= \tfrac{8}{7}v^7+\tfrac{8}{5}v^5+\tfrac{8}{3}v^3+8v+4\ln|v-1|-4\ln|v+1|+C$$
$$= \tfrac{8}{7}x^{\frac{7}{8}}+\tfrac{8}{5}x^{\frac{5}{8}}+\tfrac{8}{3}x^{\frac{3}{8}}+8x^{\frac{1}{8}}+4\ln\left|x^{\frac{1}{8}}-1\right|-4\ln\left|x^{\frac{1}{8}}+1\right|+C$$

Question 2 (5 points)

Verify all your answers in question 1 with Maple.

a)

$$\mathrm{Int}\left(\frac{\arctan(\mathrm{sqrt}(x))}{\mathrm{sqrt}(x)}, x\right) = \mathrm{int}\left(\frac{\arctan(\mathrm{sqrt}(x))}{\mathrm{sqrt}(x)}, x\right);$$

$$\int \frac{\arctan(\sqrt{x})}{\sqrt{x}}\,\mathrm{d}x = 2\sqrt{x}\,\arctan(\sqrt{x}) - \ln(1+x)$$

b)

$$\mathrm{Int}(\sin(3\cdot x)\cdot \exp(5\cdot x), x) = \mathrm{int}(\sin(3\cdot x)\cdot \exp(5\cdot x), x);$$

$$\int \sin(3\,x)\,e^{5\,x}\,\mathrm{d}x = -\frac{3}{34}\cos(3\,x)\,e^{5\,x} + \frac{5}{34}\sin(3\,x)\,e^{5\,x}$$

c)

$$Int\left(\sec(x)^6, x\right) = int\left(\sec(x)^6, x\right);$$

$$\int \sec(x)^6\,\mathrm{d}x = \frac{1}{5}\,\frac{\sin(x)}{\cos(x)^5} + \frac{4}{15}\,\frac{\sin(x)}{\cos(x)^3} + \frac{8}{15}\,\frac{\sin(x)}{\cos(x)}$$

$with(student):$

$changevar\left(u = \tan(x), Int\left(\sec(x)^6, x\right), u\right);$

$value(\%);$

$Int\left(\sec(x)^6, x\right) = subs(u = \tan(x), \%);$

$$\int \left(1+u^2\right)^2\,\mathrm{d}u$$

$$u + \frac{1}{5}\,u^5 + \frac{2}{3}\,u^3$$

$$\int \sec(x)^6\,\mathrm{d}x = \tan(x) + \frac{1}{5}\tan(x)^5 + \frac{2}{3}\tan(x)^3$$

d)

$$Int\left(\frac{3\cdot x^3 + 9\cdot x^2 + 12\cdot x + 16}{x^4 + 4\cdot x^2}, x\right) = int\left(\frac{3\cdot x^3 + 9\cdot x^2 + 12\cdot x + 16}{x^4 + 4\cdot x^2}, x\right);$$

$$\int \frac{3\,x^3 + 9\,x^2 + 12\,x + 16}{x^4 + 4\,x^2}\,\mathrm{d}x = \frac{5}{2}\arctan\left(\frac{1}{2}\,x\right) + 3\ln(x) - \frac{4}{x}$$

e)

$$Int\left(\frac{x^4 + 4\cdot x^3 + 19\cdot x^2}{x^2 + 4\cdot x + 20}, x\right) = int\left(\frac{x^4 + 4\cdot x^3 + 19\cdot x^2}{x^2 + 4\cdot x + 20}, x\right)$$

$$\int \frac{x^4 + 4\,x^3 + 19\,x^2}{x^2 + 4\,x + 20}\,\mathrm{d}x = \frac{1}{3}\,x^3 - x + 2\ln\left(x^2 + 4\,x + 20\right) + 3\arctan\left(\frac{1}{4}\,x + \frac{1}{2}\right)$$

f)

$$Int\left(\frac{x^{\left(\frac{1}{8}\right)}}{x^{\left(\frac{1}{4}\right)}-1}, x\right) = int\left(\frac{x^{\left(\frac{1}{8}\right)}}{x^{\left(\frac{1}{4}\right)}-1}, x\right)$$

$$\int \frac{x^{1/8}}{x^{1/4}-1}\,\mathrm{d}x = \frac{8}{7}x^{7/8} + \frac{8}{5}x^{5/8} + \frac{8}{3}x^{3/8} + 8\,x^{1/8} + 4\ln\left(x^{1/8} - 1\right) - 4\ln\left(x^{1/8} + 1\right)$$