MATHEMATICS 201-105-RE
Linear Algebra
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XVIII – The Simplex Method

1. Use the simplex method to solve the following linear programming problems.

a) Maximize: $f = 3x + 10y$
Subject to: $14x + 7y \le 35$
$5x + 5y \le 50$
$x \ge 0,\ y \ge 0$

b) Maximize: $f = 2x + 3y$
Subject to: $x + 2y \le 10$
$x + y \le 7$
$x \ge 0,\ y \ge 0$

c) Maximize: $f = 2x + y$
Subject to: $-x + y \le 2$
$x + 2y \ge 10$
$3x + y \le 15$
$x \ge 0,\ y \ge 0$

d) Maximize: $f = 20x + 12y + 12z$
Subject to: $x + z \le 40$
$x + y \le 30$
$y + z \le 40$
$x \ge 0,\ y \ge 0, z \ge 0$

e) Maximize: $f = 3x + 2y$
Subject to: $x - 10y \le 10$
$-x + y \le 40$
$x \ge 0,\ y \ge 0$

f) Maximize: $f = 3x + 12y$
Subject to: $2x + y \le 120$
$x + 4y \le 200$
$x \ge 0,\ y \ge 0$

g) Minimize: $z = 3x + y$
Subject to: $4x + y \ge 11$
$3x + 2y \ge 12$
$3x + y \ge 6$
$x \ge 0,\ y \ge 0$

h) Minimize: $g = 2x + 10y$
Subject to: $2x + y \ge 11$
$x + 3y \ge 11$
$x + 4y \ge 16$
$x \ge 0,\ y \ge 0$

i) Minimize: $g = 8x + 7y + 12z$
Subject to: $x + y + z \ge 3$
$y + 2z \ge 2$
$x \ge 2$
$x \ge 0,\ y \ge 0, z \ge 0$

j) Minimize: $g = 40x + 90y + 30z$
Subject to: $x + 2y + z \ge 16$
$x + 5y + 2z \ge 18$
$2x + 5y + 3z \ge 38$
$x \ge 0,\ y \ge 0, z \ge 0$

2. Consider the following problem: Maximize $f = 3x + 2y$
Subject to $x + 2y \le 16$
$x + y \le 10$
$2x + y \le 16$
$x \ge 0, y \ge 0$

a) Solve the problem using the simplex method.
b) Identify all constraints that are binding.
c) How many solutions does this problem have?
d) Find the optimal solution if the second constraint is changed to $x + y \le 8$.

3. A produce wholesaler has determined that it takes ½ hour of labor to sort and pack a crate of tomatoes and that it takes $1\frac{1}{4}$ hours to sort and pack a crate of peaches. The crate of tomatoes weighs 60 pounds and the crate of peaches weighs 50 pounds. The wholesaler has 2500 hours of labor available each week and can ship 120000 pounds per week. If the profits are $1 per crate of tomatoes and $2 per crate of peaches, how many crates of each should be sorted, packed, and shipped to maximize overall profit? What is the maximum profit?

4. The total advertising budget for a firm is $200 000. The following table gives the costs per ad package for each of the medium and the number of exposures per ad package (with all numbers in thousands).

	Medium 1	Medium 2	Medium 3
Cost/package	10	4	5
Exposures/package	3100	2000	2400

If the maximum numbers of medium 1, medium 2 and medium 3 packages that can be purchased are 18, 10 and 12 respectively, how many of each ad packages should be purchased to maximize the number of ad exposures?

5. An investor has of to $250 000 to invest in three types of investments. Type A pays 8% annually and has a risk factor of 0. Type B pays 10% annually and has a risk factor of 0.06. Type C pays 14% annually and has a risk factor of 0.10. To have a well-balanced portfolio, the investor imposes the following conditions. The average risk factor should be no greater than 0.05. Moreover, at least one-fourth of the total portfolio is to be allocated to Type A investments and at least one-fourth of the portfolio is to be allocated to Type B investments. How much should be allocated to each type of investment to obtain a maximum return?

6. A small company produces two products, I and II, at three facilities, A, B, and C. They have orders for 2000 of product I and 1200 of product II. The production capacity and cost per week to operate each facility are summarized in the following table.

	A	B	C
I	200	200	400
II	100	200	100
Cost/week	$1000	$3000	$4000

How many weeks should each facility operate to fill the company's orders at a minimum cost, and what is the minimum cost?

7. A political candidate wishes to use a combination of radio and TV advertisements in her campaign. Research has shown that each 1-minute spot on TV reaches 0.9 million people and that each 1 minute on radio reaches 0.6 million. The candidate feels she must reach 63 million people, and she must buy at least 90 minutes of advertisements. How many minutes of each medium should be used if TV costs $500 per minute, radio costs $100 per minute, and the candidate wishes to minimize costs?

Answers

1. a) Max of 50 when $x = 0$ and $y = 5$
   b) Max of 17 when $x = 4$ and $y = 3$
   c) Max of 11 when $x = 4$ and $y = 3$
   d) Max of 780 when $x = 15$, $y = 15$ and $z = 25$
   e) No solution
   f) Max of 600 either when $x = 0$ and $y = 50$, or when $x = 40$ and $y = 40$
   g) Min of 9 when $x = 2$ and $y = 3$
   h) Min of 32 when $x = 16$ and $y = 0$
   i) Min of 28 when $x = 2$, $y = 0$ and $z = 1$
   j) Min of 480 when $x = 0$, $y = 0$ and $z = 16$
2. a) Max of 26 when $x = 6$ and $y = 4$
   b) Constraints $x + y \leq 10$ and $2x + y \leq 16$ are binding
   c) One
   d) Max of 24 when $x = 8$ and $y = 0$
3. 500 tomatoes, 1800 peaches; P = 4100
4. Medium 1 = 10, Medium 2 = 10, Medium 3 = 12
5. #100 000 of type A, $62 500 of type B, $87 500 of type C; Maximum return: $26 500
6. A = 12 weeks, B = 0 weeks, C = 0 weeks; costs = $12 000
7. 105 minutes on radio, nothing on TV. Minimum cost $10 500.