

## MATHEMATICS 201-105-RE

Linear Algebra

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# XVII – Linear Programming

- Sketch the graph of the solution of the given system of linear inequalities.
  - $x \leq 2$   
 $y \leq 4$   
 $x \geq 0$   
 $y \geq 0$
  - $x + y \leq 1$   
 $-x + y \leq 1$   
 $y \geq 0$
  - $-3x + 2y < 6$   
 $x + 4y > -2$   
 $2x + y < 3$
  - $x \geq 1$   
 $x - 2y \leq 3$   
 $3x + 2y \geq 9$   
 $x + y \leq 6$
- Derive a set of inequalities to describe the given region.
  - Rectangular region with vertices at  $(2,1)$ ,  $(5,1)$ ,  $(5,7)$ , and  $(2,7)$ .
  - Triangular region with vertices at  $(0,0)$ ,  $(5,0)$ , and  $(2,3)$ .
- Find the minimum or maximum values of the given objective function, subject to the indicated constraints.
  - Objective function:  $z = 4x + 9y$   
Constraints:  $x + 5y \leq 200$   
 $2x + 3y \leq 134$   
 $x \geq 0, y \geq 0$
  - Objective function:  $z = 12x + 48y$   
Constraints:  $x + 3y \geq 3$   
 $2x + 3y \geq 5$   
 $2x + y \geq 3$   
 $x \geq 0, y \geq 0$
  - Objective function:  $z = 3x + 4y$   
Constraints:  $x + y \leq 6$   
 $2x + y \geq 10$   
 $y \leq 4$   
 $x \geq 0, y \geq 0$
  - Objective function:  $z = 4x + 3y$   
Constraints:  $2x + 3y \geq 6$   
 $3x - 2y \leq 9$   
 $x + 5y \leq 20$   
 $x \geq 0, y \geq 0$
  - Objective function:  $z = x$   
Constraints:  $2x + y \leq 28$   
 $2x + 3y \leq 60$   
 $4x + y \leq 48$   
 $x \geq 0, y \geq 0$
  - Objective function:  $z = 4x + y$   
Constraints:  $x + 2y \leq 40$   
 $x + y \geq 30$   
 $2x + 3y \geq 72$   
 $x \geq 0, y \geq 0$

- g) Objective function:  $z = 50x + 35y$   
 Constraints:  $8x + 9y \leq 7200$   
 $8x + 9y \geq 5400$   
 $x \geq 0, y \geq 0$
- h) Objective function:  $z = 2.5x + y$   
 Constraints:  $3x + 5y \leq 15$   
 $5x + 2y \leq 10$   
 $x \geq 0, y \geq 0$
- i) Objective function:  $z = -x + 2y$   
 Constraints:  $x \leq 10$   
 $x + y \leq 7$   
 $x \geq 0, y \geq 0$
- j) Objective function:  $z = 3x + 4y$   
 Constraints:  $x + y \leq 1$   
 $2x + y \geq 4$   
 $x \geq 0, y \geq 0$

4. Determine the  $t$  values such that the objective function has a maximum value at the indicated vertex.

Objective function:  $z = x + ty$

Constraints:  $x + 2y \leq 4$

$x - y \leq 1$

$x \geq 0, y \geq 0$

a)  $(0,0)$

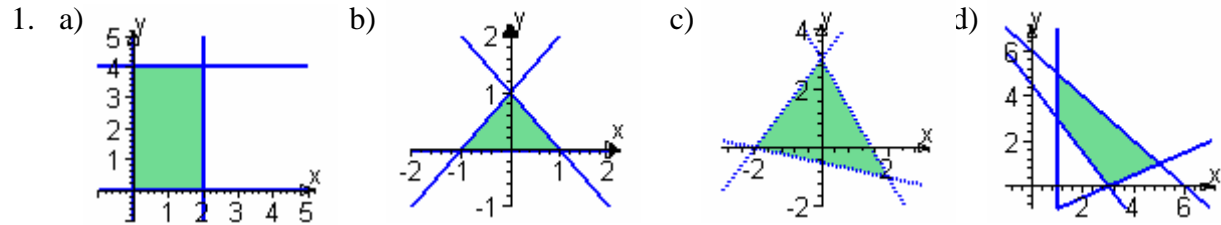
b)  $(1,0)$

c)  $(2,1)$

d)  $(0,2)$

5. A merchant plans to sell two models of home computers at cost of \$250 and \$400, respectively. The \$250 model yields a profit of \$45 and the \$400 model yields a profit of \$50. The merchant estimates that the total monthly demand will not exceed 250 units. Find the number of units of each model that should be stocked in order to maximize profit. Assume that the merchant does not want to invest more than \$70000 in computer inventory.
6. An experiment involving learning in animals requires placing white mice and rabbits into separate, controlled environments, E1 and E2. The maximum amount of time available in E1 is 500 minutes and the maximum amount of time in E2 is 700 minutes. The white mice must spend 10 minutes in E1 and 25 minutes in E2, while the rabbits must spend 15 minutes in E1 and 10 minutes in E2. Find the maximum possible number of animals that can be used in the experiment, and find the number of white mice and the number of rabbits that can be used.
7. A trucking firm ships the container of two companies, A and B. Each container from company A weighs 20 kilograms and is 0.2 cubic meter in volume. Each container from company B weighs 25 kilograms and is 0.3 cubic meter in volume. The trucking firm charges company A \$2.20 for each container shipped and charges company B \$3.00 for each container shipped. If one of the firm's trucks cannot carry more than 18 500 kilograms and cannot hold more than 200 cubic meter, how many container from companies A and B should a truck carry to maximize the shipping charges?
8. Two foods contain only proteins, carbohydrates, and fats. Food A costs \$12 per kilogram, and contains 30% protein and 50% carbohydrates. Food B costs \$15 per kilogram, and contains 20% protein and 75% carbohydrates. What combination of these two foods provides at least 10 kilogram of protein, 25 kilograms of carbohydrates and 2.5 kilograms of fat at the lowest cost?

## Answers



2. a)  $2 \leq x \leq 5$  and  $1 \leq y \leq 7$       b)  $2x \leq 3y$ ,  $y \leq -x + 5$  and  $y \geq 0$
3. a) Max of 382 when  $x = 10$  and  $y = 38$ , Min of 0 when  $x = 0$  and  $y = 0$   
 b) Min of 36 when  $x = 3$  and  $y = 0$ , No max  
 c) Max of 22 when  $x = 2$  and  $y = 4$ , Min of 0 when  $x = 0$  and  $y = 0$   
 d) Max of 27 when  $x = 3$  and  $y = 5$ , Min of 8 when  $x = 2$  and  $y = 0$   
 e) Max of 12 when  $x = 12$  and  $y = 0$ , Min of 0 when  $x = 0$  and  $y = 0$   
 f) Max of 160 when  $x = 40$  and  $y = 0$ , Min of 104 when  $x = 24$  and  $y = 8$   
 g) Max of 45000 when  $x = 900$  and  $y = 0$ , Min of 21000 when  $x = 0$  and  $y = 600$   
 h) Max of 5 on the segment from  $(2,0)$  to  $(\frac{45}{19}, \frac{20}{19})$ , Min of 0 when  $x = 0$  and  $y = 0$   
 i) Max of 14 when  $x = 0$  and  $y = 7$ , Min of -7 when  $x = 7$  and  $y = 0$   
 j) Feasible set is empty, so there are no max or mins.
4. a) No values of  $t$       b)  $t < -3$       c)  $-3 < t < 6$       d)  $t > 6$
5. 200 units of model costing \$250 and 50 units of model costing \$400, for a maximum profit of \$11500.
6. A maximum of 40 animals, 20 mice and 20 rabbits.
7. 300 containers from company A and 550 containers from company B.
8. 20 kilograms of food A and 20 kg of food B.