MATHEMATICS 201-105-RE
Linear Algebra
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XV - Spanning Sets and Linear Independence

1. Determine if the vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$ can be written as linear combinations of the vectors in $S = \{(2,-1,3),(5,0,4)\}$.
   a) $\vec{u} = (0,-5,7)$ b) $\vec{v} = (16,-\frac{1}{2},\frac{27}{2})$ c) $\vec{w} = (3,6,-2)$

2. Determine if the vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$ can be written as linear combinations of the vectors in $S = \{(2,0,7),(2,4,5),(2,-12,13)\}$.
   a) $\vec{u} = (4,-20,24)$ b) $\vec{v} = (-1,0,0)$ c) $\vec{w} = (6,24,9)$

3. Determine whether the given set *S* spans $\mathbb{R}^2$. If the set does not span $\mathbb{R}^2$, give a geometrical description of the subspace that it does span.
   a) $S = \{(2,3),(-3,2)\}$
   b) $S = \{(-3,-4)\}$
   c) $S = \{(3,-4),(-6,8)\}$

4. Determine whether the given set *S* spans $\mathbb{R}^3$. If the set does not span $\mathbb{R}^3$, give a geometrical description of the subspace that it does span.
   a) $S = \{(2,-1,3),(4,1,-2),(8,-1,4)\}$
   b) $S = \{(1,-2,3),(3,4,-1)\}$
   c) $S = \{(2,-1,3),(4,-2,6),(-6,3,-9)\}$
   d) $S = \{(1,2,3),(3,2,1),(1,3,1)\}$

5. Determine whether the given set *S* spans P_2.
   a) $S = \{5, x^2+5, 5x+1\}$
   b) $S = \{x-1, x+1, 10\}$
   c) $S = \{x^2-x+1, 2x^2+5x-3, -x^2+8x-6\}$

6. Determine whether the set S is linearly independent or dependent.

a) $S = \{(1,-3),(2,4)\}$

b) $S = \{(4,1),(2,2)\}$

c) $S = \{(3,-3),(0,0)\}$

d) $S = \{(1,1,1),(-2,-2,-2),(5,5,5)\}$

e) $S = \{(3,2,3),(4,-1,-2),(2,-1,4),(4,1,-5)\}$

f) $S = \{(3,-1,3),(1,1,1),(2,2,1)\}$

g) $S = \{(0,0,0,1),(0,0,1,1),(0,1,1,1),(1,1,1,1)\}$

h) $S = \{(1,-3,2,1),(-2,5,2,1),(3,-8,0,0),(-1,2,4,2)\}$

7. Show that the given set is linearly dependent by finding a nontrivial linear combination (of vectors in the set) whose sum is the zero vector. Then express one of the vectors in the set as a linear combination of the other vectors in the set.

a) $S = \{(2,-3),(1,4),(-2,5)\}$

b) $S = \{(1,-3,2),(5,2,1),(3,8,-3)\}$

8. For what values of t are the following sets linearly independent?

a) $S = \{(t,1,1),(1,t,1),(1,1,t)\}$

b) $S = \{(t,t,1),(t,0,t),(1,1,t)\}$

9. Determine which of the sets in $M_{2,2}$ are linearly independent.

a) $S = \left\{\begin{bmatrix}1 & 3\\ 0 & 2\end{bmatrix},\begin{bmatrix}3 & -2\\ 0 & 4\end{bmatrix},\begin{bmatrix}1 & -8\\ 0 & 0\end{bmatrix}\right\}$

b) $S = \left\{\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix},\begin{bmatrix}0 & 4\\ -3 & 0\end{bmatrix},\begin{bmatrix}1 & -8\\ 0 & 0\end{bmatrix}\right\}$

c) $S = \left\{\begin{bmatrix}2 & 1\\ -1 & 3\end{bmatrix},\begin{bmatrix}4 & 0\\ -2 & 3\end{bmatrix},\begin{bmatrix}-1 & 2\\ 0 & 6\end{bmatrix},\begin{bmatrix}0 & 1\\ 1 & 1\end{bmatrix}\right\}$

10. Determine which of the sets in P_2 are linearly independent.

a) $S = \{1-x^2, 1+x^2, 2x+1\}$

b) $S = \{x^2+x+1, x+2, x^2-3x-7\}$

c) $S = \{x^2+x-1, x^2-x+1, x^2+2x+1\}$

Answers

1. a) $\vec{u} = 5(2,-1,3) - 2(5,0,4)$ b) $\vec{v} = \frac{1}{2}(2,-1,3) + 3(5,0,4)$
   c) $\vec{w}$ cannot be written as a linear combination of the given vectors.
2. a) $\vec{u} = 7(2,0,7) - 5(2,4,5) + 0(2,-12,13)$
   b) $\vec{v}$ cannot be written as a linear combination of the given vectors
   c) $\vec{w} = -3(2,0,7) + 6(2,4,5) + 0(2,-12,13)$.
3. a) S spans $\mathbb{R}^2$
   b) S does not span $\mathbb{R}^2$ (It spans the $4x - 3y = 0$ line)
   c) S does not span $\mathbb{R}^2$ (It spans the line $4x + 3y = 0$)
4. a) S does not span $\mathbb{R}^3$ (It spans the plane $x - 16y - 6z = 0$)
   b) S does not span $\mathbb{R}^3$ (It spans the plane $x - y - z = 0$)
   c) S does not span $\mathbb{R}^3$ (It spans the line $\frac{x}{2} = \frac{y}{-1} = \frac{z}{3}$)
   d) S spans $\mathbb{R}^3$
5. a) S spans P_2
   b) S does not span P_2
   c) S does not span P_2
6. a) Linearly independent
   b) Linearly independent
   c) Linearly dependent
   d) Linearly dependent
   e) Linearly dependent
   f) Linearly independent
   g) Linearly independent
   h) Linearly dependent
7. a) $13(2,-3) - 4(1,4) + 11(-2,5) = (0,0)$, $(2,-3) = \frac{4}{13}(1,4) - \frac{11}{13}(-2,5)$
   b) $2(1,-3,2) - (5,2,1) + (3,8,-3) = (0,0,0)$, $(1,-3,2) = \frac{1}{2}(5,2,1) - \frac{1}{2}(3,8,-3)$
8. a) $t \neq -2,1$ b) $t \neq -1,0,1$
9. a) Linearly dependent
   b) Linearly independent
   c) Linearly independent
10. a) Linearly independent
    b) Linearly dependent
    c) Linearly independent