

## MATHEMATICS 201-105-RE

Linear Algebra

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# XII - Lines in $\mathbb{R}^3$

1. Find, if possible, an equation for the line in

- i) vector form
- ii) parametric form
- iii) symmetric form

a) Passing through  $P(5,-1,2)$  and parallel to the vector  $\vec{u} = (2, 7, -4)$ .

b) Passing through the points  $P(-2,3,5)$  and  $Q(4,1,-3)$ .

c) Passing through the points  $P(1,0,6)$  and  $Q(-3,0,7)$ .

d) Passing through the points  $P(3,-1,8)$  and  $Q(-6,2,-16)$ .

e) Passing through the point  $P(1,5,1)$  and parallel to 
$$\begin{cases} x = -1 + 4t \\ y = 2 - t \\ z = 5 + 3t \end{cases}, \quad t \in \mathbb{R}.$$

f) Passing through the point  $P(2,4,1)$  and parallel to  $\frac{x+5}{2} = \frac{2-y}{3} = 4z$ .

g) Passing through the point  $P(3,-2,6)$  and parallel to

$$(x, y, z) = (-2, 1, 4) + t(-1, 2, 2), \quad t \in \mathbb{R}.$$

h) Passing through  $P(-1,3,5)$  and perpendicular to the lines  $L_1: \frac{x+3}{2} = y+5 = 2z$  and

$$L_2: (x, y, z) = (-1, 2, 2) + t(1, 2, -5), \quad t \in \mathbb{R}$$

i) Passing through  $P(2,-1,4)$  and perpendicular to the lines  $L_3: \frac{x}{2} = \frac{y-3}{2} = \frac{z+1}{3}$  and

$$L_4: \begin{cases} x = -3 + t \\ y = 2 - 5t \\ z = 4 + t \end{cases}, \quad t \in \mathbb{R}.$$

j) Passing through  $P(2,-3,5)$  and perpendicular to the  $xz$ -plane.

k) Passing through  $P(-1,1,5)$  and the intersection of the lines

$$L_1: (x, y, z) = (1, 2, 3) + t(-2, 7, -1), \quad t \in \mathbb{R} \quad \text{and} \quad L_2: \begin{cases} x = 4 + t \\ y = 5 + 10t \\ z = 6 + 2t \end{cases}, \quad t \in \mathbb{R}.$$

2. Consider the following lines.

$$L_1 : \frac{x-1}{2} = \frac{y}{-2} = -2z$$

$$L_2 : (x, y, z) = (0, 2, -1) + t(1, 1, 0), \quad t \in \mathbb{R}$$

$$L_3 : \begin{cases} x = -1 + 2t \\ y = 3 \\ z = t \end{cases}, \quad t \in \mathbb{R}$$

$$L_4 : \frac{x-3}{2} = y-3 = \frac{2-z}{3}$$

$$L_5 : (x, y, z) = (3, -2, 1) + t(4, -4, -1), \quad t \in \mathbb{R}$$

- Find the angle between the line  $L_1$  and  $L_2$ ,  $L_1$  and  $L_3$ ,  $L_1$  and  $L_4$ ,  $L_1$  and  $L_5$ .
  - Find which lines are parallel or perpendicular with  $L_1$ .
  - Find, if possible, the intersection of the lines  $L_2$  and  $L_4$ ,  $L_3$  and  $L_4$ ,
  - Determine the distance from each line to the point  $P(1, 2, -3)$ .
  - Find the point  $Q$  on each line that is closest to the point  $P(-1, 2, 2)$ .
  - Determine the distance between the pair of lines  $L_1$  and  $L_5$ , and  $L_2$  and  $L_3$ .
  - For each of the points  $A(5, 3, 3)$ ,  $B(1, 1, 1)$  and  $C(-1, 2, 2)$ , determine to which line they belong.
3. Consider the lines  $L_1 : \frac{x-d}{3} = \frac{2y-4}{3} = \frac{16z+20}{b}$  and  $L_2 : \frac{3x+6}{a} = \frac{4-y}{2} = \frac{4z+8}{3}$ .
- Find the value of the constants  $a$  and  $b$  for which the lines are parallel.
  - Find the value of  $d$  for which the lines are equivalent.
4. Find the point  $P$  on the line  $\frac{x+1}{-1} = \frac{y}{2} = \frac{z-2}{4}$  that is closest to the origin.
5. Consider the line  $L : 5 - x = y + 4 = \frac{z+3}{2}$ .
- Find the point  $Q$  on the line  $L$  that is closest to the point  $P(8, 9, 1)$ .
  - Find the distance between the point  $P$  and the line  $L$ .
  - Find the equation of a line perpendicular to  $L$  and passing through the point  $P$ .

## Answers

$$1. \text{ a) } (x, y, z) = (5, -1, 2) + t(2, 7, -4), \quad t \in \mathbb{R} \quad \begin{cases} x = 5 + 2t \\ y = -1 + 7t, & t \in \mathbb{R} \\ z = 2 - 4t \end{cases}$$

$$\frac{x-5}{2} = \frac{y+1}{7} = \frac{2-z}{4}$$

$$\text{b) } (x, y, z) = (-2, 3, 5) + t(6, -2, -8), \quad t \in \mathbb{R} \quad \begin{cases} x = -2 + 6t \\ y = 3 - 2t, & t \in \mathbb{R} \\ z = 5 - 8t \end{cases}$$

$$\frac{x+2}{6} = \frac{3-y}{2} = \frac{5-z}{8}$$

$$\text{c) } (x, y, z) = (1, 0, 6) + t(-4, 0, 1), \quad t \in \mathbb{R} \quad \begin{cases} x = 1 - 4t \\ y = 0, & t \in \mathbb{R} \\ z = 6 + t \end{cases}$$

None, but we do have  $\frac{1-x}{4} = z-6, \quad y=0$

$$\text{d) } (x, y, z) = t(3, -1, 8), \quad t \in \mathbb{R} \quad \begin{cases} x = 3t \\ y = -t, & t \in \mathbb{R} \\ z = 8t \end{cases} \quad \frac{x}{3} = \frac{y}{-1} = \frac{z}{8}$$

$$\text{e) } (x, y, z) = (1, 5, 1) + t(4, -1, 3), \quad t \in \mathbb{R} \quad \begin{cases} x = 1 + 4t \\ y = 5 - t, & t \in \mathbb{R} \\ z = 1 + 3t \end{cases}$$

$$\frac{x-1}{4} = 5-y = \frac{z-1}{3}$$

$$\text{f) } (x, y, z) = (2, 4, 1) + t(2, -3, \frac{1}{4}), \quad t \in \mathbb{R} \quad \begin{cases} x = 2 + 2t \\ y = 4 - 3t, & t \in \mathbb{R} \\ z = 1 + \frac{1}{4}t \end{cases}$$

$$\frac{x-2}{2} = \frac{4-y}{3} = 4z-4$$

$$\text{g) } (x, y, z) = (3, -2, 6) + t(-1, 2, 2), \quad t \in \mathbb{R} \quad \begin{cases} x = 3 - t \\ y = -2 + 2t, & t \in \mathbb{R} \\ z = 6 + 2t \end{cases}$$

$$3-x = \frac{y+2}{2} = \frac{z-6}{2}$$

$$\text{h) } (x, y, z) = (-1, 3, 5) + t(-6, \frac{21}{2}, 3), \quad t \in \mathbb{R} \quad \begin{cases} x = -1 - 6t \\ y = 3 + \frac{21}{2}t, \\ z = 5 + 3t \end{cases} \quad t \in \mathbb{R}$$

$$\frac{x+1}{-6} = \frac{2y-6}{21} = \frac{z-5}{3}$$

$$\text{i) } (x, y, z) = (2, -1, 4) + t(17, 1, -12), \quad t \in \mathbb{R} \quad \begin{cases} x = 2 + 17t \\ y = -1 + t, \\ z = 4 - 12t \end{cases} \quad t \in \mathbb{R}$$

$$\frac{x-2}{17} = y+1 = \frac{4-z}{12}$$

$$\text{j) } (x, y, z) = (2, -3, 5) + t(0, 1, 0), \quad t \in \mathbb{R} \quad \begin{cases} x = 2 \\ y = -3 + t, \\ z = 5 \end{cases} \quad t \in \mathbb{R}$$

None, but we have  $x = 2, z = 5$

$$\text{k) } (x, y, z) = (-1, 1, 5) + t(4, -6, -1), \quad t \in \mathbb{R} \quad \begin{cases} x = -1 + 4t \\ y = 1 - 6t, \\ z = 5 - t \end{cases} \quad t \in \mathbb{R}$$

$$\frac{x+1}{4} = \frac{1-y}{6} = 5-z$$

2. a)  $90^\circ, 57^\circ, 71^\circ, 0^\circ$

b)  $L_1 \perp L_2 \quad L_1 // L_5$

c) none,  $(3, 3, 2)$

d)  $\frac{2\sqrt{3333}}{33}, \frac{3\sqrt{2}}{2}, \frac{\sqrt{345}}{5}, \frac{4\sqrt{70}}{14}, \frac{2\sqrt{6501}}{33}$

e)  $(\frac{-13}{11}, \frac{24}{11}, \frac{6}{11}), (\frac{-1}{2}, \frac{3}{2}, -1), (\frac{-1}{5}, 3, \frac{2}{5}), (\frac{12}{7}, \frac{33}{14}, \frac{55}{14}), (-1, 2, 2)$

f)  $\frac{2\sqrt{66}}{11}, \frac{2\sqrt{6}}{3}$

g)  $A \in L_3$  and  $C \in L_5$

3. a)  $a = -12, b = -9$       b)  $d = 6$

4.  $(\frac{-4}{7}, \frac{-6}{7}, \frac{2}{7})$

5. a)  $Q(2, -1, 3)$       b)  $\sqrt{140} = 2\sqrt{35}$       c)  $\frac{x-8}{6} = \frac{y-9}{10} = z+2$