MATHEMATICS 201-105-RE
Linear Algebra
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X - Cross Product

1. Let $\vec{u}=(1,-2,3)$, $\vec{v}=(4,5-1)$ and $\vec{w}=(-2,1,7)$. Compute
   a) $\vec{u}\times\vec{v}$
   b) $\vec{v}\times\vec{w}$
   c) $\vec{u}\times(\vec{v}\times\vec{w})$
   d) $(\vec{u}\times\vec{v})\times\vec{w}$
   e) $(\vec{u}\times\vec{v})\times(\vec{v}\times\vec{w})$
   f) $\vec{u}\times(\vec{v}-2\vec{w})$

2. Find a vector that is orthogonal to both $\vec{u}$ and $\vec{v}$.
   a) $\vec{u}=(1,-2,3)$, $\vec{v}=(4,5-1)$
   b) $\vec{u}=(4,-3,5)$, $\vec{v}=(-8,0,6,)$

3. Find the area of the parallelogram determined by $\vec{u}$ and $\vec{v}$.
   a) $\vec{u}=(3,0)$, $\vec{v}=(1,1)$
   b) $\vec{u}=(\frac{1}{2},-\frac{2}{3})$, $\vec{v}=(4,3)$
   c) $\vec{u}=(4,0,-3)$, $\vec{v}=(0,-2,5)$
   d) $\vec{u}=(1,-3,2)$, $\vec{v}=(-1,-1,-1)$

4. Find the area of the triangle with vertices
   a) $A(1,0)$, $B(2,3)$ and $C(6,0)$
   b) $A(6,5)$, $B(1,3)$ and $C(3,-2)$
   c) $A(1,2,3)$, $B(-1,-3,2)$ and $C(5,0,5)$

5. Find the scalar triple product $\vec{u}\bullet(\vec{v}\times\vec{w})$.
   a) $\vec{u}=(1,-2,3)$, $\vec{v}=(4,5-1)$, $\vec{w}=(-1,2,0)$
   b) $\vec{u}=(2,2,2)$, $\vec{v}=(0,-2,1)$, $\vec{w}=(2,-2,4)$

6. Let $\vec{u}\bullet(\vec{v}\times\vec{w})=2$. Find
   a) $\vec{u}\bullet(\vec{w}\times\vec{v})$
   b) $(\vec{v}\times\vec{w})\bullet\vec{u}$
   c) $\vec{w}\bullet(\vec{u}\times\vec{v})$
   d) $\vec{v}\bullet(\vec{u}\times\vec{w})$
   e) $(\vec{u}\times\vec{w})\bullet\vec{v}$
   f) $\vec{u}\bullet(\vec{w}\times\vec{w})$

7. Find the volume of the parallelepiped with sides $\vec{u}$, $\vec{v}$ and $\vec{w}$.
   a) $\vec{u}=(1,-2,3)$, $\vec{v}=(4,5-1)$, $\vec{w}=(-2,1,7)$.
   b) $\vec{u}=(1,-3,2)$, $\vec{v}=(-1,-1,-1)$, $\vec{w}=(2,-2,4)$

8. Find the volume of the tetrahedron with vertices A, B, C and D.
   a) $A(-1,4,4)$, $B(-1,-3,2)$, $C(5,0,5)$, $D(2,-3,4)$
   b) $A(3,-1,3)$, $B(2,2,-1)$, $C(1,-1,3)$, $D(2,3,4)$

9. Determine whether $\vec{u}$, $\vec{v}$ and $\vec{w}$ lie in the same plane when positioned so that their initial points coincide.
   a) $\vec{u}=(2,6,6)$, $\vec{v}=(4,5-1)$, $\vec{w}=(-2,1,7)$
   b) $\vec{u}=(1,-3,2)$, $\vec{v}=(-1,-1,-1)$, $\vec{w}=(5,-1,-2)$

10. Prove that if θ is the angle between $\vec{u}$ and $\vec{v}$, where $\vec{u}$ and $\vec{v}$ are nonorthogonal vectors in $\mathbb{R}^3$, then

$$\tan\theta=\frac{\|\vec{u}\times\vec{v}\|}{\vec{u}\bullet\vec{v}}$$

11. Prove the following identities for vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$ in $\mathbb{R}^3$.
    a) $(\vec{u}+k\vec{v})\times\vec{v}=\vec{u}\times\vec{v}$
    b) $(\vec{u}+\vec{v})\times(\vec{u}-\vec{v})=-2(\vec{u}\times\vec{v})$

Answers

1. a) $(-13,13,13)$ b) $(36,-26,14)$ c) $(50,94,46)$
   d) $(78,65,13)$ e) $(520,650,-130)$ f) $(21,39,19)$
2. a) $(-13,13,13)$ b) $(-18,-64,-24)$
3. a) 3 b) $\frac{25}{6}$ c) $10\sqrt{5}$ d) $\sqrt{42}$
4. a) $\frac{15}{2}$ b) $\frac{29}{2}$ c) $6\sqrt{5}$
5. a) 39 b) 0
6. a) –2 b) 2 c) 2 d) –2 e) –2 f) 0
7. a) 130 b) 4
8. a) $\frac{13}{2}$ b) $\frac{19}{3}$
9. a) Yes b) No
10. Since $\theta\neq 90°$, then $\|\vec{u}\|\|\vec{v}\|=\dfrac{\vec{u}\bullet\vec{v}}{\cos\theta}$. Also, since $\|\vec{u}\times\vec{v}\|=\|\vec{u}\|\|\vec{v}\|\sin\theta$ then $\|\vec{u}\times\vec{v}\|=\dfrac{\vec{u}\bullet\vec{v}}{\cos\theta}\sin\theta$ which gives us $\tan\theta=\dfrac{\|\vec{u}\times\vec{v}\|}{\vec{u}\bullet\vec{v}}$.
11. a) $(\vec{u}+k\vec{v})\times\vec{v}=\vec{u}\times\vec{v}+(k\vec{v}\times\vec{v})=\vec{u}\times\vec{v}+k(\vec{v}\times\vec{v})=\vec{u}\times\vec{v}+k\vec{0}=\vec{u}\times\vec{v}$
    b) $(\vec{u}+\vec{v})\times(\vec{u}-\vec{v})=(\vec{u}+\vec{v})\times\vec{u}-(\vec{u}+\vec{v})\times\vec{v}=\vec{u}\times\vec{u}+\vec{v}\times\vec{u}-\vec{u}\times\vec{v}-\vec{v}\times\vec{v}$
    $=\vec{0}-\vec{u}\times\vec{v}-\vec{u}\times\vec{v}-\vec{0}=-2(\vec{u}\times\vec{v})$