MATHEMATICS 201-105-RE
Linear Algebra
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Definition of a Vector Space

Let V be a set on which two operations (***vector addition*** $\oplus$ and ***scalar multiplication*** $\odot$) are defined. If the following axioms are satisfied for every $\vec{u}$, $\vec{v}$ and $\vec{w}$ in V and every scalar k and l, then V is called a ***vector space***.

Vector Addition

1. $\vec{u} \oplus \vec{v} \in V$	closure under addition
2. $\vec{u} \oplus \vec{v} = \vec{v} \oplus \vec{u}$	commutativity
3. $\vec{u} \oplus (\vec{v} \oplus \vec{w}) = (\vec{u} \oplus \vec{v}) \oplus \vec{w}$	associativity
4. V has a ***zero vector*** $\vec{0}$ such that for every $\vec{u}$ in V, $\vec{u} \oplus \vec{0} = \vec{u}$	additive identity
5. For every $\vec{u}$ in V, there is a vector in V, denoted $-\vec{u}$, such that $\vec{u} \oplus (-\vec{u}) = \vec{0}$	additive inverse

Scalar Multiplication

6. $k \odot \vec{u} \in V$	closure under scalar multiplication
7. $(k \oplus l) \odot \vec{u} = (k \odot \vec{u}) \oplus (l \odot \vec{u})$	distributivity
8. $k \odot (\vec{u} \oplus \vec{v}) = (k \odot \vec{u}) \oplus (k \odot \vec{v})$	distributivity
9. $k \odot (l \odot \vec{u}) = (kl) \odot \vec{u}$	associativity
10. $1 \odot \vec{u} = \vec{u}$	scalar identity

Test for a Subspace

If W is a subset of a vector space V, then W is a subspace of V if and only if

1. If $\vec{u}$ and $\vec{v}$ are in W, then $\vec{u} \oplus \vec{v} \in W$
2. If $\vec{u}$ is in W and k is a scalar, then $k \odot \vec{u} \in W$