

MATHEMATICS 201-105-RE

Linear Algebra

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VIII - Algebraic Vectors

1. Consider the points A(2,3), B(-1,4), C(3,-5), D(-3,3). Find the components of the following vectors.

a) \overrightarrow{AB} b) \overrightarrow{BC} c) \overrightarrow{CD} d) \overrightarrow{CB} e) \overrightarrow{DA}

2. Let $\vec{u} = (1,5)$, $\vec{v} = (-2,1)$ and $\vec{w} = (-5,-3)$. Find

a) $6\vec{u}$ b) $0\vec{u}$ c) $\vec{u} + \vec{v}$ d) $2\vec{u} - 5\vec{v}$ e) $2\vec{u} + 3\vec{v} - 7\vec{w}$

f) $\|\vec{u}\|$ g) $\|-2\vec{v}\|$ h) $\|4\vec{u} - \vec{v}\|$ i) $\frac{\vec{v}}{\|\vec{v}\|}$ j) $\left\| \frac{\vec{w}}{\|\vec{w}\|} \right\|$

k) \vec{z} if $4\vec{u} - 3\vec{z} = 5\vec{w}$ l) A unit vector in the direction of \vec{u}

3. Given the vectors $\vec{u} = (-3,2)$ and $\vec{v} = (4,3)$. Sketch

a) $\frac{1}{2}\vec{u}$ b) $-2\vec{u}$ c) $\vec{u} + \vec{v}$ d) $2\vec{u} - 3\vec{v}$

4. Consider the points A(2,3), B(-1,4), C(3,-5) and D(-3,3) given in question 1. Find

a) $\overrightarrow{AB} - \overrightarrow{CD}$ b) $\overrightarrow{BC} - \overrightarrow{AB} + \overrightarrow{CD}$ c) $5\overrightarrow{AB} - 3\overrightarrow{AC} + 2\overrightarrow{DB}$

d) $\|\overrightarrow{CB} - \overrightarrow{AB}\|$ e) $\|\overrightarrow{AD} - \overrightarrow{DA}\|$

5. Show that points A(2,1), B(5,7), C(10,14) and D(7,8) are the vertices of a parallelogram.

6. Find the components of the following vectors.

a) $\|\vec{u}\| = 5$ N30°W b) $\|\vec{u}\| = 10$ S45°E c) $\|\vec{u}\| = 8$ S60°W

7. Find the vector $\vec{a} = \vec{u} + 2\vec{v} - 4\vec{w}$, using their components, if

$\|\vec{u}\| = 5$ N40°W $\|\vec{v}\| = 7$ S50°W $\|\vec{w}\| = 9$ S40°E

8. Consider $\vec{u} = (u_1, u_2)$. Prove that $\|k\vec{u}\| = |k| \cdot \|\vec{u}\|$.

8. Prove that if we have the points $A(x_a, y_a)$ and $B(x_b, y_b)$, then $\|\overrightarrow{AB}\| = \|\overrightarrow{BA}\|$.

9. Consider the points A(2,3,-1), B(3,2,-4), C(3,0,1), D(-1,1,0). Find the components of the following vectors.

a) \overrightarrow{AB} b) \overrightarrow{BC} c) \overrightarrow{CD} d) \overrightarrow{CB} e) \overrightarrow{DA}

10. Let $\vec{u} = (1, 5, 2)$, $\vec{v} = (-2, -4, 1)$ and $\vec{w} = (6, -5, 3)$. Find

- a) $6\vec{u}$ b) $\vec{w} - \vec{u}$ c) $3\vec{u} + 2\vec{v}$ d) $\frac{2\vec{u} - 5\vec{v}}{6}$ e) $2\vec{u} + 3\vec{v} - 7\vec{w}$
- f) $\|\vec{u}\|$ g) $\|-2\vec{v}\|$ h) $\|4\vec{u} - \vec{v}\|$ i) $\frac{\vec{v}}{\|\vec{v}\|}$ j) $\left\| \frac{\vec{w}}{\|\vec{w}\|} \right\|$
- k) \vec{z} if $4\vec{u} - 3\vec{z} = 5\vec{w}$ l) A unit vector in the direction of \vec{u}

11. Given the vectors $\vec{u} = (-3, 2, 2)$ and $\vec{v} = (1, 0, -2)$. Sketch

- a) $\frac{1}{2}\vec{u}$ b) $-2\vec{u}$ c) $\vec{u} + \vec{v}$ d) $3\vec{u} - 2\vec{v}$

12. Prove that if $\vec{v} = (a, b, c)$ then $\frac{\vec{v}}{\|\vec{v}\|}$ is a unit vector.

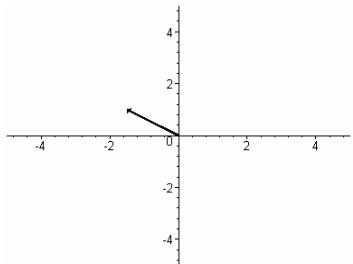
13. Let $\vec{u} = (2, -3, 0, 3, 7)$, $\vec{v} = (1, -2, 3, 4, 5)$ and $\vec{w} = (4, 0, -3, 2, -1)$. Find

- a) $6\vec{u}$ b) $5\vec{v}$ c) $\vec{u} + \vec{v}$ d) $\vec{u} - \frac{1}{2}\vec{v}$ e) $3\vec{u} - 4\vec{v} + 2\vec{w}$
- f) $\|\vec{u}\|$ g) $\|3\vec{v}\|$ h) $\|4\vec{u} - \vec{v}\|$ i) $\frac{\vec{v}}{\|\vec{v}\|}$ j) $\left\| \frac{\vec{w}}{\|\vec{w}\|} \right\|$
- k) \vec{z} if $2\vec{u} + 8\vec{z} = \vec{w}$ l) A unit vector in the direction of \vec{u}

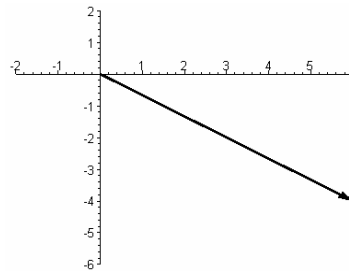
Answers

1. a) $(-3,1)$ b) $(4,-9)$ c) $(-6,8)$ d) $(-4,9)$ e) $(5,0)$
 2. a) $(6,30)$ b) $(0,0)$ c) $(-1,6)$ d) $(12,5)$ e) $(31,34)$
 f) $\sqrt{26}$ g) $2\sqrt{5}$ h) $\sqrt{397}$ i) $(\frac{-2\sqrt{5}}{5}, \frac{\sqrt{5}}{5})$ j) 1
 k) $(\frac{29}{3}, \frac{35}{3})$ l) $(\frac{\sqrt{26}}{26}, \frac{5\sqrt{26}}{26})$

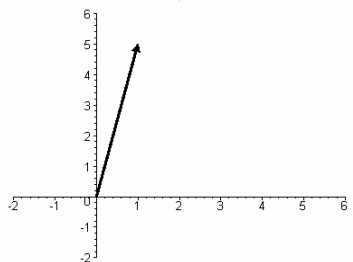
3. a)



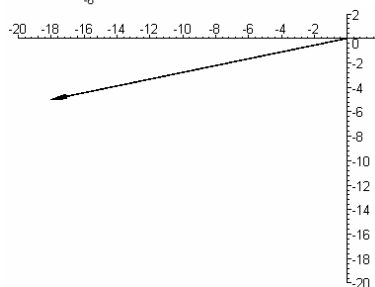
b)



c)



d)



4. a) $(3,-7)$ b) $(1,-2)$ c) $(-14,31)$ d) $\sqrt{65}$ e) 0

5. $\overline{AB} = (3,6) = \overline{DC}$ and $\overline{BC} = (5,7) = \overline{AD}$

6. a) $(\frac{-5}{2}, \frac{5\sqrt{3}}{2})$ b) $(5\sqrt{2}, -5\sqrt{2})$ c) $(-4\sqrt{3}, -4)$

7. $\|\vec{a}\| = 43.3$ N58.9°W

8. $\|k\vec{u}\| = \|k(u_1, u_2)\| = \|(ku_1, ku_2)\| = \sqrt{(ku_1)^2 + (ku_2)^2} = \sqrt{k^2u_1^2 + k^2u_2^2}$
 $= \sqrt{k^2(u_1^2 + u_2^2)} = |k|\sqrt{u_1^2 + u_2^2} = |k|\|\vec{u}\|$

9. $\|\overline{AB}\| = \|(x_b - x_a, y_b - y_a)\| = \sqrt{(x_b - x_a)^2 + (y_b - y_a)^2} = \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2}$
 $= \|(x_a - x_b, y_a - y_b)\| = \|\overline{BA}\|$

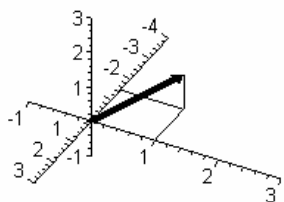
10. a) $(1,-1,-3)$ b) $(0,-2,5)$ c) $(-4,1,-1)$ d) $(0,2,-5)$ e) $(3,2,-1)$

11. a) $(6,30,12)$ b) $(5,-10,1)$ c) $(-1,7,8)$ d) $(2,5,\frac{-1}{6})$ e) $(-46,33,-14)$

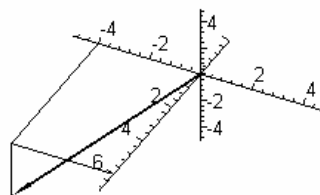
f) $\sqrt{30}$ g) $2\sqrt{21}$ h) $\sqrt{661}$ i) $(\frac{-2\sqrt{21}}{21}, \frac{-4\sqrt{21}}{21}, \frac{\sqrt{21}}{21})$

j) 1 k) $(\frac{-26}{3}, 15, \frac{-7}{3})$ l) $(\frac{\sqrt{30}}{30}, \frac{\sqrt{30}}{6}, \frac{\sqrt{30}}{15})$

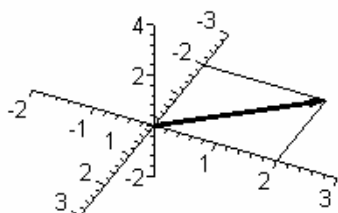
12. a)



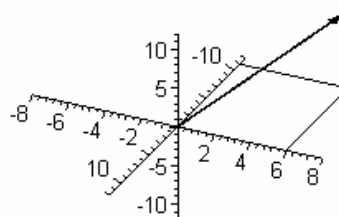
b)



c)



d)



$$13. \frac{\|\vec{v}\|}{\|\vec{v}\|} = \left\| \frac{1}{\sqrt{a^2 + b^2 + c^2}} (a, b, c) \right\| = \left\| \left(\frac{a}{\sqrt{a^2 + b^2 + c^2}}, \frac{b}{\sqrt{a^2 + b^2 + c^2}}, \frac{c}{\sqrt{a^2 + b^2 + c^2}} \right) \right\| = \sqrt{\frac{a^2}{a^2 + b^2 + c^2} + \frac{b^2}{a^2 + b^2 + c^2} + \frac{c^2}{a^2 + b^2 + c^2}}$$

$$= \sqrt{\frac{a^2 + b^2 + c^2}{a^2 + b^2 + c^2}} = \sqrt{1} = 1$$

$$14. \text{ a) } (12, -18, 0, 18, 42) \quad \text{b) } (5, -10, 15, 20, 25) \quad \text{c) } (3, -5, 3, 7, 12) \quad \text{d) } \left(\frac{3}{2}, -2, \frac{-3}{2}, 1, \frac{9}{2}\right)$$

$$\text{e) } (10, -1, -18, -3, -1) \quad \text{f) } \sqrt{71} \quad \text{g) } 3\sqrt{55} \quad \text{h) } \sqrt{751}$$

$$\text{i) } \left(\frac{\sqrt{55}}{55}, \frac{-2\sqrt{55}}{55}, \frac{3\sqrt{55}}{55}, \frac{4\sqrt{55}}{55}, \frac{\sqrt{55}}{11}\right) \quad \text{j) } 1 \quad \text{k) } \left(0, \frac{3}{4}, \frac{-3}{8}, \frac{-1}{2}, \frac{-15}{8}\right)$$

$$\text{l) } \left(\frac{2\sqrt{71}}{71}, \frac{-3\sqrt{71}}{71}, 0, \frac{3\sqrt{71}}{71}, \frac{7\sqrt{71}}{71}\right)$$