

MATHEMATICS 201-105-RE

Linear Algebra

Martin Huard

Winter 2006

VI - Applications

1. An encyclopedia salesman works for a company that offers three different grades of bindings for its encyclopedias: standard, deluxe and leather. For each set he sells, he earns a commission that is based on the set's binding grade. One week he sells one standard, one deluxe, and two leathers sets and makes \$675 in commission. The next week he sells two standard, one deluxe, and one leather set for a \$600 commission. The third week he sells one standard, two deluxe, and one leather set, earning \$625 in commission. Translate the given information into a system of linear equations, then solve the system to find how much commission the salesman earn on a set of encyclopedias in each binding grade.
2. A small school has 100 students who occupy three classrooms: rooms A, B, C. After the first period of the school day, half the students in room A move to room B, one-fifth of the students in room B move to room C, and one-third of the students in room C move to room A. Nevertheless, the total number of students in each room is the same for each period. How many students occupy each room?
3. A furniture manufacture produces 3 models of desks. Each model uses different quantities of wood, plywood and veneer. The quantities are given in the following table.

	M_1	M_2	M_3
Wood	12	16	14
Plywood	1.5	2	1.8
Veneer	0.8	0.6	1.2

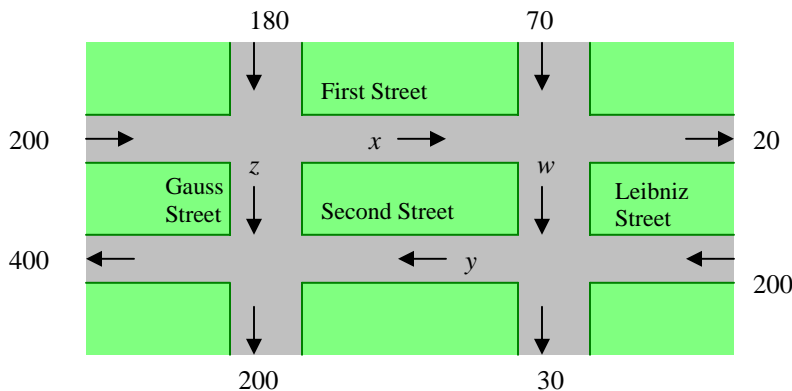
The wood is in units of length and the measure for plywood and veneer is in units of area.

- a) The company has in reserve the following quantities: 530 units of wood, 66.9 units of plywood and 31.8 units of veneer. How many desks of each model can these reserves build?
- b) The company has orders for 29 desks of model 1, 55 desks of model 2 and 43 of model 3. What additional quantities of each material must the company order to fulfill the order?
- c) The time to make each desk, in minutes of work, per person is given in the following table along with the weekly spare time in each of the shops.

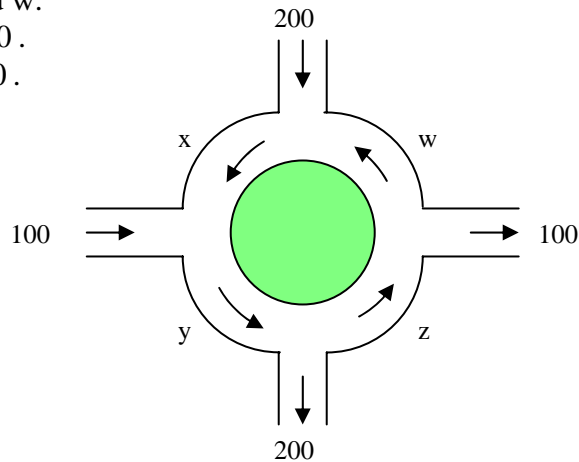
	M_1	M_2	M_3	Spare time
Wood	75	90	85	5010
Plywood	45	50	65	3170
Veneer	50	65	90	4050

How many desks of each model can the company manufacture each week with the spare time?

4. Suppose you are the manager of a music store. At the end of one week you are going over receipts for the previous week's sales. One thousand and thirty five compact discs were sold. One type of compact disc sold for \$9.99, another one for \$14.99 and a third one for \$19.99. The total compact disc receipts were \$14 464.65. The cash register that was supposed to record the number of each type of compact disc sold malfunctioned. Can you recover the information? What if, based on your experience from previous weeks, you assume that you sold twice as many compact discs at \$9.99 then at \$19.99? If so, how many of each type of compact disc were sold?
5. A father and his three daughters are celebrating their birthday today. Knowing that the father's age is twice the sum of the age of his daughters, and thirteen times the difference in age between the two youngest daughters, and also knowing that the age of the eldest is three times the difference in age of her sisters, and that a quarter of the age of the eldest is equal to a third the age of the second oldest, determine the age of each person on this anniversary day.
6. A small corporation borrowed \$775 000 to expand its product line. Some of the money was borrowed at 8%, some at 9%, and some at 10%. How much was borrowed at each rate if the annual interest owed was \$67 000 and the amount borrowed at 8% was four times the amount borrowed at 10%.
7. An inheritance of \$40 000 was divided among three investments yielding \$3500 in interest per year. The interest rates of the three investments were 7%, 9% and 11%. Find the amount placed in each investment if the second and third were \$3000 and \$5000 less than the first, respectively.
8. A diagram of a section of the street network in a city is shown in the figure, where the arrows indicate one-way streets. The numbers on the diagram show how many cars enter or leave this section of the city via the indicated street in a certain one-hour period. The variables x , y , z and w represent the number of cars that travel along the portions of First, Second, Gauss and Leibniz Streets during this period. Find x , y , z and w , assuming that none of the cars involved in this problem stop or park on any of the streets shown in the figure.



9. The flow of traffic (in vehicles per hour) through a network of streets is shown below.
- Solve this system for x , y , z and w .
 - Find the traffic flow when $w = 0$.
 - Find the traffic flow when $x = 0$.



10. Suppose a government study estimated that the probability of successive generations of a rural family remaining in a rural area was 0.7, and the probability of successive generations of an urban family remaining in an urban area was 0.9. Assuming a Markov chain applies to these facts, find the steady state vector.
11. A local business A has two competitors, B and C . Initially the probability that a customer patronizes A , B , or C is 0.2, 0.6 and 0.2 respectively. Suppose A initiates an advertising campaign to improve its business and finds the following transition matrix to describe the effect.

$$\begin{bmatrix} 0.7 & 0.4 & 0.4 \\ 0.2 & 0.4 & 0.4 \\ 0.1 & 0.2 & 0.2 \end{bmatrix}$$

- If A runs the advertising campaign for 2 weeks, find the probability of a customer patronizing each business.
 - Find the steady-state vector for this market, that is, the long-range share of the market that each business can expect if the transition matrix holds.
12. Three supermarkets serve a certain section of a city. During the upcoming year, supermarket A is expected to retain 80% of its customers, lose 5% of its customers to supermarket B and lose 15% to supermarket C . Supermarket B is expected to retain 90% of its customers and lose 5% of its customers to each of supermarkets A and C . Supermarket C is expected to retain 75% of its customers, lose 10% to supermarket A and lose 15% to supermarket B . The current market shares of supermarkets A , B and C are 0.4, 0.3 and 0.3 respectively.
- What share of the market will be held after one year?
 - What share of the market will be held after two years, assuming that the trend continues?
 - Find the steady-state vector for this market, assuming that the trend continues.

13. Three neighbors have backyard vegetable gardens. Neighbor A grows tomatoes, neighbor B grows corn, and neighbor C grows lettuce. They agree to divide their crops among themselves as follows: A gets $\frac{1}{2}$ of the tomatoes, $\frac{1}{3}$ of the corn, and $\frac{1}{4}$ of the lettuce. B gets $\frac{1}{3}$ of the tomatoes, $\frac{1}{3}$ of the corn, and $\frac{1}{4}$ of the lettuce. C gets $\frac{1}{6}$ of the tomatoes, $\frac{1}{3}$ of the corn, and $\frac{1}{2}$ of the lettuce. What prices should the neighbors assign to their respective crops if the equilibrium condition of a closed economy system is to be satisfied, and if the lowest-priced crop is to have a price of \$100?
14. A system composed of two industries, coal and steel, has the following inputs.
- To produce one dollar's worth of output, the coal industry requires \$0.10 of its own product and \$0.80 of steel.
 - To produce one dollar's worth of output, the steel industry requires \$0.10 of its own product and \$0.20 of coal.
- Find the input-output matrix for this system. Then solve for the output matrix, where the external demand for coal is \$10 000 and for steel \$20 000.
15. A small community includes a farmer, baker, and grocer. To produce 1 dollar worth of goods, a farmer needs \$0.40 of its own good, \$0.30 from the baker and \$0.20 from the grocer, the baker needs \$0.50 from the farmer, nothing from himself and \$0.20 from the grocer, while the grocer needs \$0.50 of the farmer, \$0.30 of the baker and nothing from himself. If the external demand is \$1000 for each, what must be their output?
16. Find the equation of the parabola (if possible) passing through the given points.
- (1,0), (2,3), (-4,45)
 - (2,-1), (-3,-6), (4,-13)
 - (-3,4), (2,1), (-3,5)
 - (-2,-7), (2,1), (3,3)
17. Find the equation of the third degree polynomial $y = ax^3 + bx^2 + cx + d$ passing through the points (1,4), (2,21), (-1,-6) and (-2,-23).

Answers

1. $x + y + 2z = 675$ He earns \$125 for the standard, \$150 for the deluxe and \$200
 $2x + y + z = 600$ for the leather set.
 $x + 2y + z = 625$
2. There are 20 students in room A, 50 in room B and 30 in room C.
3. a) They can build 9 desks of M_1 , 15 desks of M_2 and 13 desks of M_3 .
b) 1300 units of wood, 164 units of plywood and 76 units of veneer.
c) They can build 20 desks of M_1 , 22 desks of M_2 and 18 desks of M_3 .
4. a) No, there is an infinite amount of solutions
b) Yes. 420 compact discs at \$9.99, 405 compact discs at \$14.99 and 210 compact discs at \$19.99.
5. Father : 52 and the daughters 12, 9 and 5.
6. \$366 666.67 at 8%, \$316 666.67 at 9% and \$91 666.67 at 10%.
7. \$16 000 at 7%, 13 000 at 9% and \$11 000 at 11%.
8. $x = t - 50$, $y = t + 170$, $z = 430 - t$ and $w = t$
9. a) $x = 200 + t$, $y = 300 + t$, $z = 100 + t$ and $w = t$
b) $x = 200$, $y = 300$, $z = 100$ and $w = 0$
c) Impossible for x to be zero, because then $w = -200$, and we assume a positive flow.
10. $(\frac{1}{4}, \frac{3}{4})$
11. a) A customer will patronize A, B and C with probability 0.538, 0.308 and 0.154.
b) $(\frac{4}{7}, \frac{2}{7}, \frac{1}{7})$
12. a) The market shares will be 36.5%, 33.5% and 30%.
b) The market shares will be 33.88%, 36.48% and 29.65%.
c) $(\frac{1}{4}, \frac{1}{2}, \frac{1}{4})$
13. Tomatoes should be at \$120, corn at \$100 and lettuce at \$106.67.
14. $C = \begin{bmatrix} 0.1 & 0.2 \\ 0.8 & 0.1 \end{bmatrix}$ The output should be \$20 000 for coal and \$40 000 for steel.
15. The output should be \$8 622.05 for the farmer, \$4685.04 for the baker and \$3 661.42 for the grocer.
16. a) $2x^2 - 3x + 1$ b) $-x^2 + 3$ c) No solution
d) No parabola but the line $y = 2x - 3$
17. $2x^3 + 3x - 1$