

MATHEMATICS 201-105-RE

Linear Algebra

Martin Huard

Winter 2006

Properties of Matrices

Properties of Matrix Arithmetic

1. $A + B = B + A$ (Commutative law for addition)
2. $A + (B + C) = (A + B) + C$ (Associative law for addition)
3. $A(BC) = (AB)C$ (Associative law for multiplication)
4. $A(B + C) = AB + AC$ (Left distributive law)
5. $(A + B)C = AC + BC$ (Right distributive law)
6. $a(A + B) = aA + aB$
7. $(a + b)A = aA + bA$
8. $a(bA) = (ab)A$
9. $a(AB) = (aA)B$

Properties of Zero Matrices

1. $A_{m \times n} + 0_{m \times n} = 0_{m \times n} + A_{m \times n} = A_{m \times n}$
2. $A_{m \times n} - A_{m \times n} = 0_{m \times n}$
3. $0_{m \times n} - A_{m \times n} = -A_{m \times n}$
4. $A_{m \times r} 0_{r \times n} = 0_{m \times n}$
 $0_{m \times r} A_{r \times n} = 0_{m \times n}$

Properties of the Transpose

1. $(A^T)^T = A$
2. $(A + B)^T = A^T + B^T$
3. $(kA)^T = kA^T$
4. $(AB)^T = B^T A^T$

Properties of Identity Matrices

1. $A_{m \times n} I_{n \times n} = A_{m \times n}$
2. $I_{m \times m} A_{m \times n} = A_{m \times n}$

Properties of the Trace

1. $tr(A + B) = tr(A) + tr(B)$
2. $tr(kA) = k \cdot tr(A)$
3. $tr(AB) = tr(BA)$

Exponents

Definition: If A is a square matrix then for nonnegative integers n ,

$$A^0 = I \qquad A^n = \underbrace{AA \cdots A}_{n \text{ factors}}$$

If A is invertible, then, for negative integers, we have

$$A^{-n} = (A^{-1})^n = \underbrace{A^{-1}A^{-1} \cdots A^{-1}}_{n \text{ factors}}$$

Properties: 1. $A^r A^s = A^{r+s}$

2. $(A^r)^s = A^{rs}$

Properties of Inverses

If A and B are invertible matrices, then:

1. A^{-1} is invertible with $(A^{-1})^{-1} = A$
2. kA is invertible with $(kA)^{-1} = \frac{1}{k} A^{-1}$
3. A^T is invertible with $(A^T)^{-1} = (A^{-1})^T$
4. AB is invertible with $(AB)^{-1} = B^{-1}A^{-1}$
5. A^n is invertible with $(A^n)^{-1} = (A^{-1})^n$
6. $A^{-1} = \frac{1}{\det(A)} \text{adj}(A)$

Matrices as products of Elementary Matrices

1. $A^{-1} = E_k E_{k-1} \cdots E_3 E_2 E_1$
2. $A = E_1^{-1} E_2^{-1} E_3^{-1} \cdots E_k^{-1}$

Properties of determinants

1. $\det(A^T) = \det(A)$
2. $\det(kA) = k^n \det(A)$
3. $\det(AB) = \det(A)\det(B)$
4. $\det(A^{-1}) = \frac{1}{\det(A)}$ (if A is invertible)